INTERSECTED LOW-DENSITY PARITY-CHECK AND CONVOLUTIONAL CODES

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Key words to describe this work: Low-density parity-check (LDPC) codes, convolutional codes, EXIT charts

Key Results: Proposed a new code which compares favourably with other sparse-graph block codes. The graph structure allows analysis by EXIT charts.

How does the work advance the state-of-the-art?: A gain of 1.5dB is found over a standard deep-space coding scheme while still maintaining some of the computational advantages of the original scheme.

Motivation (Problems addressed): To try and get the best of the stream and block coding worlds.

1 Introduction

In current communication systems it is common to use a serial concatenation of a Reed-Solomon (R-S) code with a convolutional code (SCRSCC) [3, 5]. In decoding, the convolutional code is decoded first to a maximum likelihood codeword and this is then used as an input to the R-S decoder. Information is not used effectively in this process, the convolutional code is returning a hard decision. A R-S decoder is in general not capable of using soft information.

Large block codes can approach the Shannon Limit [2]. In general, these suffer from non-linear time encoding algorithms and the need for a complete block to be received before decoding can be attempted (this is a problem for low-speed telemetry links). To keep encoders close to linear time and allow some errors to be corrected on the fly, it is useful to keep the structure of a convolutional code combined with a high rate block code.

Convolutional codes with a soft-input soft-output decoder [7] have been shown to perform well in concatenated coding schemes [1]. This paper looks at replacing the R-S code component of the SCRSCC with a low-density parity-check (LDPC) code [8]. A LDPC code used alone is a good code.

Iterative decoding [6] will be used. We will use extrinsic information transfer (EXIT) charts [9] to analyse a code's performance in terms of properties of the messages being passed during decoding.

2 Code Construction

To maintain a simple graph structure we will look at the intersection of a LDPC code with a convolutional code (ILDPCCC). An example factor graph showing the constraints satisfied is shown in figure 1. Decoding is carried out by belief propagation on the factor graph [6].

With the LDPC component kept to be high rate, the majority of the encoding can happen in linear time with the convolutional code. The final few bits will be treated as a general linear block code.

3 Regular LDPC component code

LDPC codes are often "regular" – this means that every variable node has the same number of links connected to it. The degree of the variable nodes, j, is the number of check nodes each is connected to. When regular $j = 3$ LDPC codes are used on their own, good performance is obtained. So a ILDPCCC code was simulated with a $j = 3$ LDPC component code, figure 2.

Strong codes are often not good constituent codes in an iterative decoding scheme. When the waterfall region is steep (as with an LDPC code) the constituent decoder gives an "all-or-nothing" answer; this does not help iterative decoding. The strong LDPC code of the previous simulation was replaced by a weak $j = 1$ LDPC code. An S-random interleaver [4] was used to avoid an error-floor due to the local structure of both the LDPC and convolutional codes. In simulations, figure 2, a 0.5dB gain in performance was found.

4 Comparison with a deep-space standard

A standard for deep-space communications is a rate 1/2 convolutional code serially concatenated with (255,223) R-S codes [3]. This was compared to a ILDPCCC code, figure 3. For comparison a regular LDPC code is also shown. It can be seen that a gain of 1.5dB over the deep-space standard is achieved with the ILDPCCC and that the intersection code compares favourably with a non-optimized sparse graph block code.

5 EXIT chart analysis

The behaviour of the iterative decoding can be shown using an EXIT chart. An EXIT chart plots the transfer of information back and forth between two constituent parts of the decoding graph during decoding. For each part we seek to obtain the function relating the information input and information output in an iteration. For this function to exist, it is necessary that the graph section be loop-free. In figure

1 the graph has been shown split into two loop-free sections; the left section consists of the check nodes and the right section consists of the variable nodes and the convolutional code. The transfer function of the right section depends on the channel noise level; the left section does not. An advantage of ILDPCCC codes over turbo codes [1] is that the form of the transfer functions can be altered by adjusting the degree sequences of the nodes.

Following [10] approximate forms for the transfer function of the variable node and check nodes were used. The transfer function of the convolutional code was simulated and then a tanh function fitted to it, figure 4(a).

An example EXIT chart is shown in figure 4(b). In the limit of large block size a code will decode if a swath exists between the two curves. The channel noise level was altered by binary division to find the threshold where the curves just intersect.

Thresholds were calculated for the $j=1$ and $j=3$ ILDPCCC codes from section 3 and are shown in figure 2. Optimization of the degree sequences was carried out and a further threshold improvement of 0.05dB over the regular $j=1$ case could be found.

References

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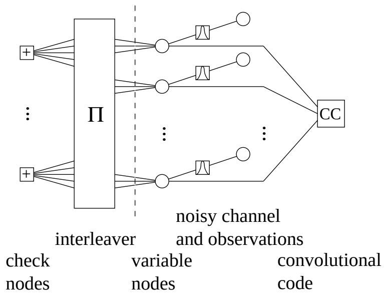


Figure 1: The factor graph of a ILDPCCC code, shown with a Gaussian noise channel. The point of message analysis, section 5, is shown with a dotted line.

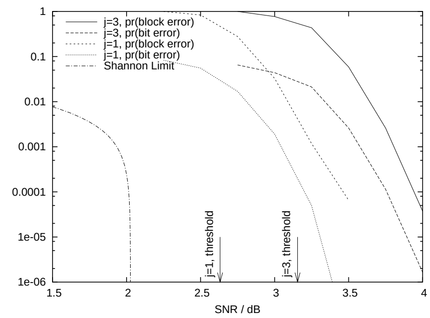


Figure 2: Convolutional code ($N=3000$, $R=3/4$, $m=7$) intersected with two regular LDPC codes ($R=0.9$ for both)

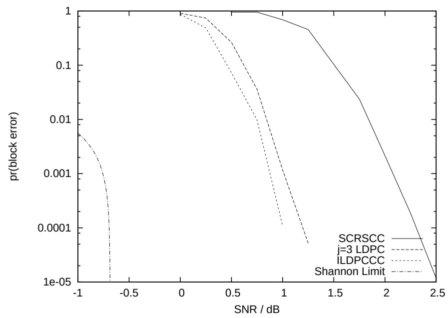
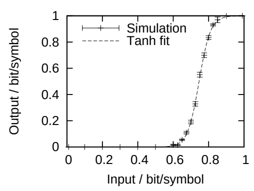
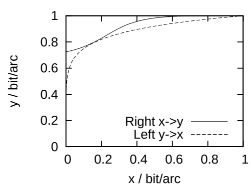


Figure 3: A comparison of a ILDPCCC with SCRSCC and LDPC codes. All codes have $N=4092$ and $R=0.4360$. The two constituent convolutional codes are identical.



(a) Single $m=7$, $R=3/4$ convolutional code



(b) $j=1$ ILDPCCC at threshold

Figure 4: EXIT charts