Learning Proof and Questioning Lies

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ABSTRACT. I developed a course in ‘Guessing and proving’. One goal was to learn how to critique everyday arguments: to transfer skills from the mathematics class to the social world. Students said that thanks to the class they no longer accept what they read in the newspaper. I then wondered why mathematical demonstration applies so well to the social world. The reason, no pun intended, lies in the origin of demonstrative argument: It arose partly from the prevalence of rhetoric in Athenian democratic culture. If, after reading this paper, you also find it painful to read the newspaper, don’t sue me.

What Use is Learning Proof?

‘Mathematics teaches thinking’ – this mantra is chanted especially about double-column proofs, whose decline is lamented by the mathematician Barry Simon:

> ...those who have survived those darned dual columns understand something about argumentation and logic. They can more readily see through the faulty reasoning so often presented in the media and by politicians. (1998)

However, I have not noticed particularly acute social analysis from mathematicians. A refrain sung from the temples of learning and Mammon is that the ‘ever-changing global economy of the 21st century’ demands that people accept inferior public transport, health care, or education. Barry Simon, in spite of his training in proof, accepts the dog-eat-dog assumption of the globalisers,[1] thereby deducing the need for proof:

> In the global economy, our young people will be in competition with young people the world over...For a large number of jobs in our technologically based world, a solid scientific and mathematical training is essential and our foreign competitors are beating us there. (1998)

As Simon’s glib reasoning suggests, learning proof in the traditional way is no guarantee that the skills will transfer to other domains, especially nonmathematical ones. The traditional applications of proof, typically all mathematical, are similar to one another, so the ideas common to all the applications include much besides the essence of proof. Thus the student cannot easily abstract the essential, transferable ideas (Figure 1). To teach for transfer, you have to apply the ideas in widely differing contexts.[2]

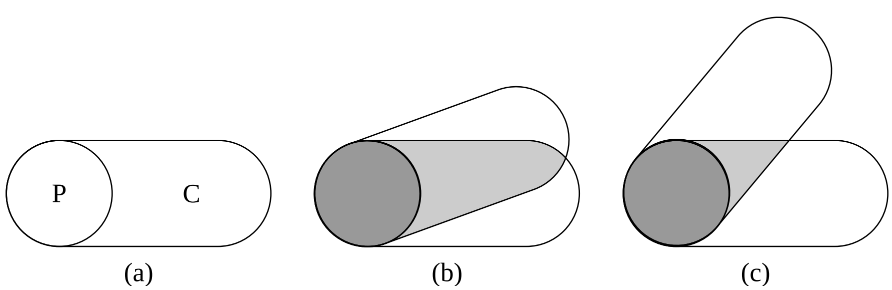

Figure 1. *Why knowledge from traditional courses does not transfer. (a) Ideas used in an application, showing contingent, contextual ideas (C) and proof ideas (P). (b) Traditional course with two applications. The contexts are similar, so the intersection of the two applications (shaded area) includes much more than the essence of proof (darker shaded area). (c) Teaching for transfer. Here the contexts are diverse, so the intersection is mostly the essence of proof.*

Teaching Proof So That The Skill Transfers

So last year I developed a nontraditional course on 'Guessing and proving' for A-level mathematics students in the Cambridge area (A-level is the last two years of English secondary school). Inspiration came from Fawcett's (1938) year-long geometry course, but I wanted to include mathematics beyond geometry and in only 20 hours. We began with:

> Into how many regions do five planes divide space? (Polya 1954, pp. 43–52)

Students learnt the courage to guess (32 is interesting). They learnt to distinguish educated from random guesses and to prove (or disprove!) their guesses. We schematised our understanding of proof with the fox, goose, and corn puzzle (a farmer must carry fox, goose, and corn across a river without the fox eating the goose or the goose eating the corn). A proof tart, we found, is baked from five ingredients:

1. *Theorems (T).* Claim for which a deductive argument is given: 'You can carry the fox, goose, and corn across safely.'
2. *Postulate (P).* Explicit, unjustified premise: 'Geese eat corn.'
3. *Proof step (S).* Deduction justified using postulates and previous deductions and theorems: 'You can first carry the goose across, since the fox is safe with the corn.'
4. *Definition (D).* Introduces a shorthand.
5. *Assumption (A).* Unspoken, implicit postulate: 'The goose will not waddle away while you are in the rowboat.'

We read a newspaper article (Cohen 1999) with particularly dubious arguments – admittedly a competitive field – and began by discussing two definitions:

> ...'genetically modified' ... – that is, rendered more productive, more hardy, less vulnerable to fungal and viral pests through scientific alteration, including the addition of genes.
>
> ...the Monsanto Corporation's Roundup Ready soybean seeds – gene-altered to resist fungus and weeds – ...

One student pointed out that the first definition should include 'rendered more profitable to large corporations'. The second definition fared no better: Roundup Ready seeds are gene-altered to resist Monsanto's weed killer rather than fungus. The students then gave hell to the next few sentences, in which I have labelled and italicised some ingredients, as the students learnt to do:[3]

> Mr. Lugar, who would like to see scientific *testing of genetically modified crops in Europe,*^{T} confessed to being amazed. His argument is simple. *The population of the world will probably grow to nine billion from six billion by 2050.*^{P} *Available acreage for planting has already been identified.*^{P} So, unless food productivity is increased – *which will not happen without scientific intervention*^{P} – *people are going to go hungry.*^{T}

They questioned the postulate that available acreage 'has been identified', and wondered about the definition of 'identified'. They spotted the implicit assumptions that present techniques cannot feed nine billion people, that GM techniques increase food productivity, and that other techniques would not.[4] The class found so many ingredients, and missing ingredients, that I could hardly shut them up. We spent two hours discussing the article, watching it crumble before the students' developing skepticism. This method of analysis creates a flood of questions rather than answers: 'I told them stories and I tried to strengthen their natural contrariness...: *the best education consists in immunising people against systematic attempts at education*' (Feyerabend 1987, p. 316, his italics).

Why This Method Transfers to Everyday Life

At first I was surprised how well the mathematical skills transfered to the social world. Then I learnt from Geoffrey Lloyd how proof arose partly because of the democratic culture of ancient Athens,[5] which included huge and frequent juries (Socrates was tried by a jury of 501). Ordinary Athenians needed skill in rhetoric. The spread of rhetorical teaching brought a reaction:

> Both Plato and Aristotle repeatedly contrast the merely persuasive with the incontrovertibly true... it is Aristotle who offers the first full philosophical analysis of the conditions that have to be fulfilled for a conclusion to be said to have been demonstrated. (Lloyd 1992, p. 51)

In short, demonstration, this rhetorical ace of trumps, was designed in and for the social world. By using it to analyse social arguments, by happy accident we had reversed history and recovered a lost tradition.

Practice

Every newspaper contains cannon fodder, or you can enjoy critiquing this statement:

> Most legal scholars say the professors have a pretty weak case [that NATO committed war crimes in Kosovo], noting that accidental civilian deaths caused by NATO bombs fail to meet the commonly accepted standard for war crimes. (Truehart 2000)

The students said that the proof course should be taught throughout the United Kingdom and that they no longer believe what they read in the newspaper,[6] the happy result of a historically orientated mathematics course.

ENDNOTES

1. For pro-globalisation, see Friedman (1999); for contra, see Hahnel (1999).
2. Bransford, Brown, and Cocking (1999, ch. 3) give a valuable discussion of transfer. See especially the references on how ‘contrasting cases’ enhances transfer (p. 48).
3. Why are there no ‘A’ (assumption) symbols in the labelling?
4. We should also have wondered how to define ‘productivity’!
5. See Lloyd (1979, pp. 59ff, 240ff; 1992; 1996, ch. 4) for details and nuances of this argument.
6. Fred Flener has found many of Fawcett’s former students and has interviewed them for a book he is writing. Sixty years after the course, Fawcett’s students remember it as the best they ever took!

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