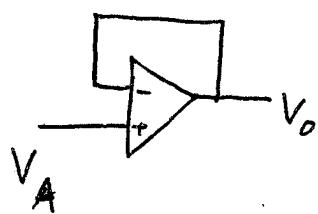


2000

Paper 3

2000 (3)

1)

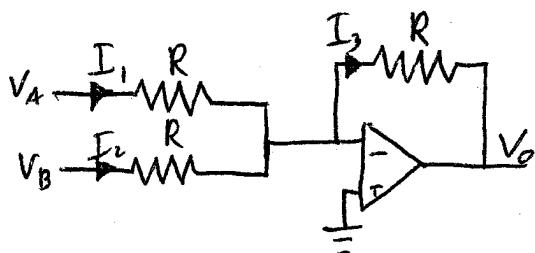


$$V_+ = V_A$$

$$V_- = V_o$$

Perfect op-amp $\Rightarrow V_+ \approx V_-$

$$\Rightarrow \underline{\underline{V_o = V_A}}$$



Perfect op-amp $\Rightarrow V_+ \approx V_-$

$$V_+ = 0$$

$$V_A - V_- = I_1 R = V_A$$

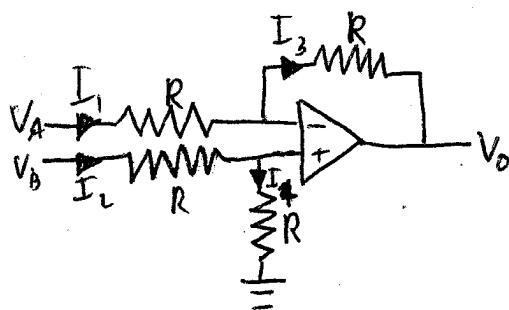
$$V_B - V_- = I_2 R = V_B$$

$$V_- - V_o = I_3 R = -V_o$$

$$I_1 + I_2 = I_3$$

$$\Rightarrow \frac{V_A}{R} + \frac{V_B}{R} = -\frac{V_o}{R}$$

$$\Rightarrow \underline{\underline{V_o = -(V_A + V_B)}}$$



Perfect op-amp $\Rightarrow V_+ \approx V_-$

$$I_1 = I_3$$

$$I_2 = I_4$$

$$V_B - V_+ = I_2 R$$

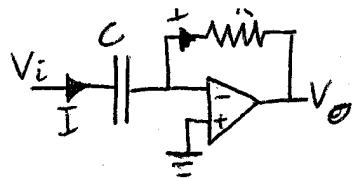
$$V_+ = I_4 R = I_2 R \Rightarrow V_B = 2V_+$$

$$V_A - V_- = I_1 R$$

$$V_- - V_o = I_3 R = I_1 R \Rightarrow V_A + V_o = 2V_-$$

$$\Rightarrow V_B = V_A + V_o$$

$$\Rightarrow \underline{\underline{V_o = V_B - V_A}}$$

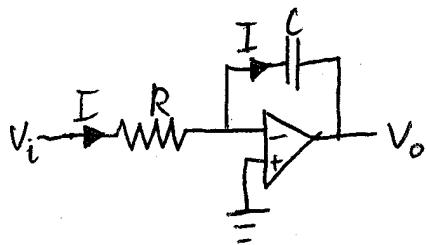


$$V_- \approx V_+ = 0$$

$$-V_o = IR = R \frac{dQ}{dt}$$

$$Q = CV_i$$

$$\Rightarrow V_o = -RC \frac{dV_i}{dt}$$



$$V_- \approx V_+ = 0$$

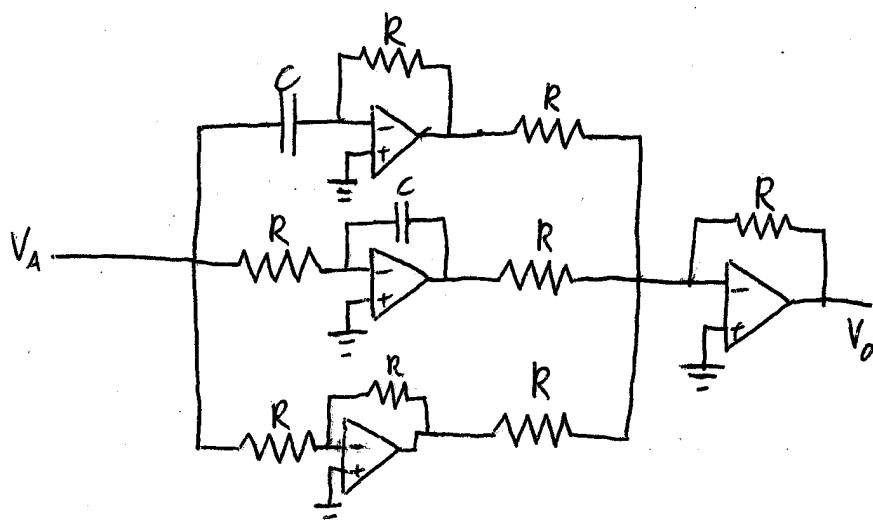
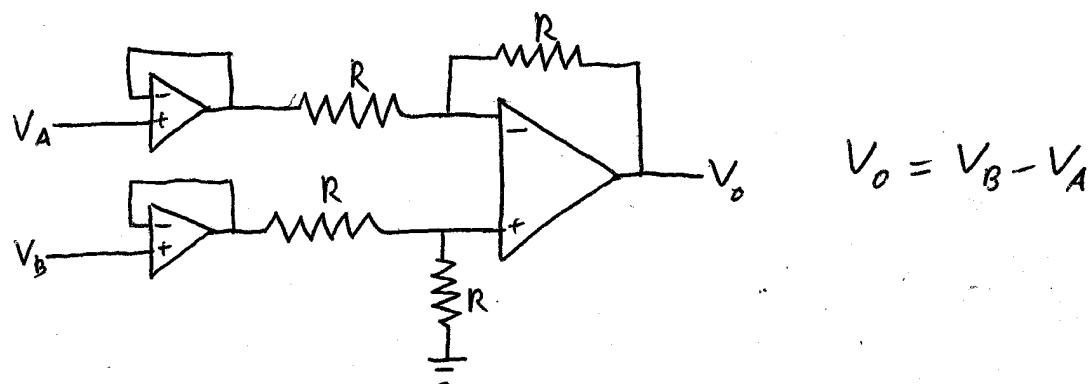
$$V_i = IR$$

$$Q = -CV_o = SI dt$$

$$-CV_o = \int \frac{V_i}{R} dt$$

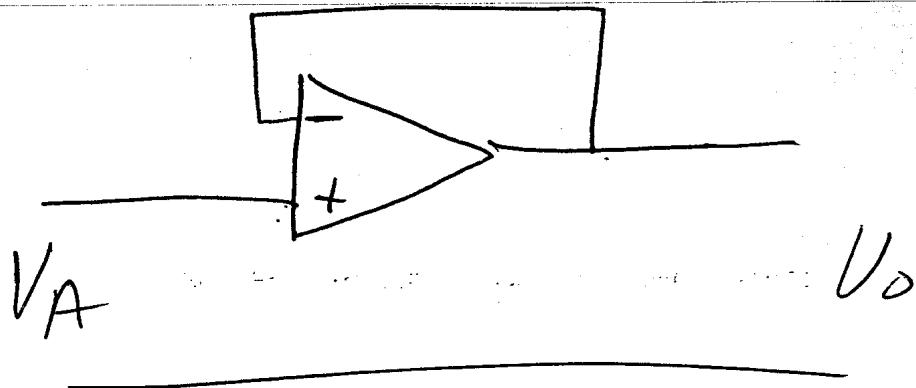
$$V_o = - \int \frac{V_i}{RC} dt$$

To measure voltage difference from 2 high output impedance sources

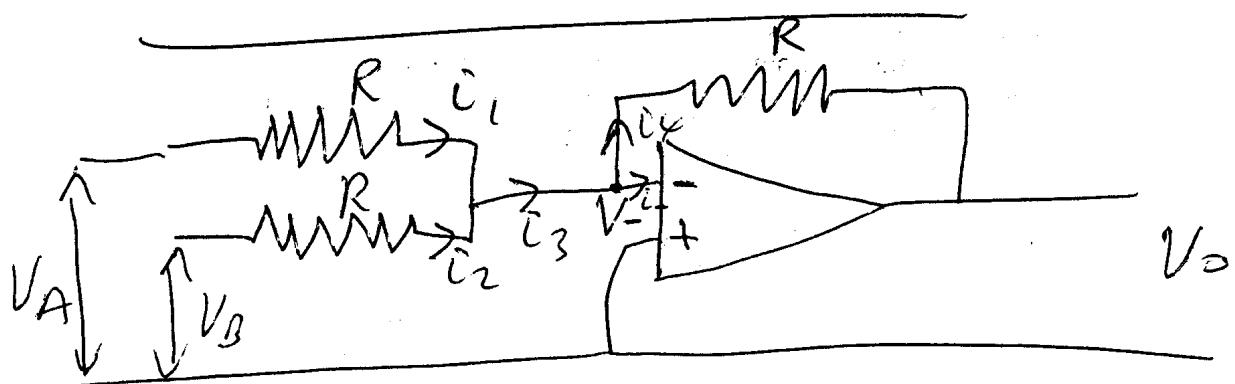


$$V_o = K_1 V_A + K_2 \frac{dV_A}{dt} + K_3 \int V_A dt$$

Qn 1



$$V_O = V_A$$



$$V_+ = 0 = V_- \quad i_- = 0$$

$$i_1 + i_2 = i_3 = i_4$$

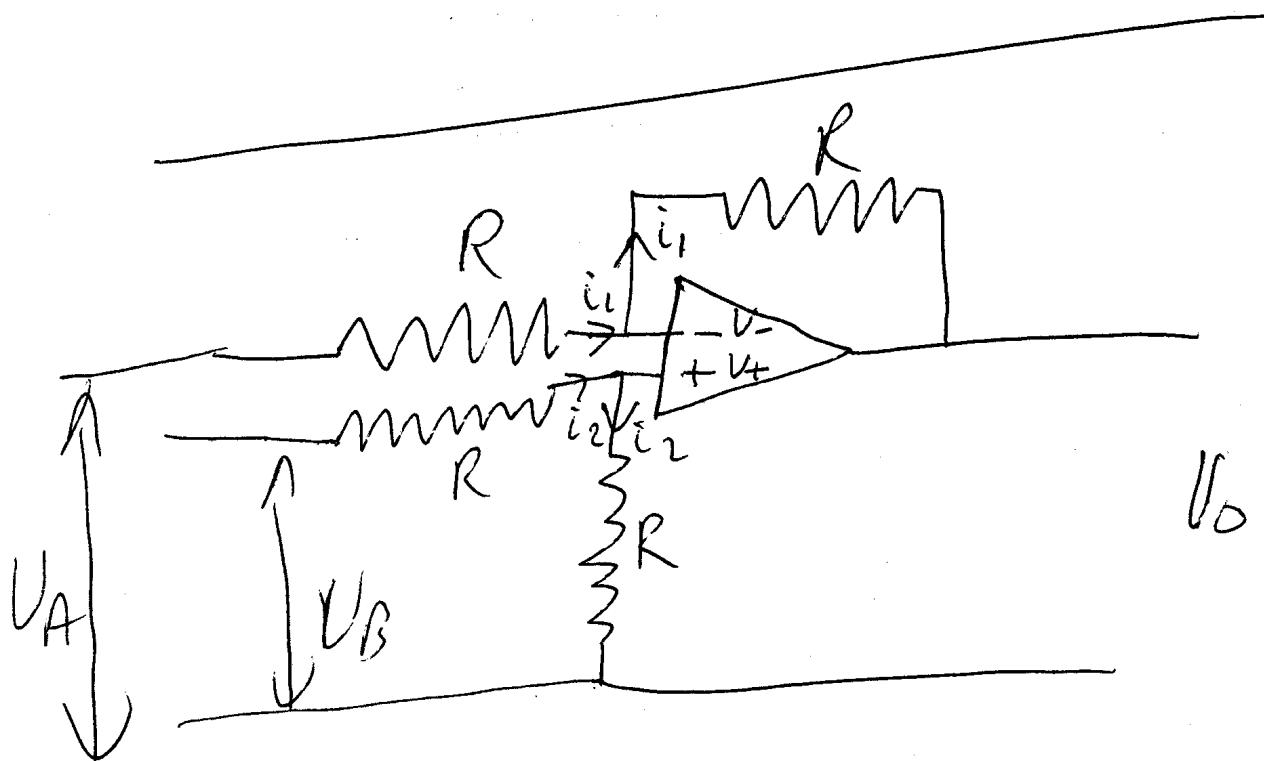
$$\frac{V_A - V_-}{R} = i_1 = \frac{V_A}{R}$$

$$\frac{V_B - V_-}{R} = i_2 = \frac{V_B}{R}$$

$$i_4 = \frac{V_- - V_O}{R} = -\frac{V_O}{R}$$

$$\frac{V_A}{R} + \frac{V_B}{R} = -\frac{V_o}{R}$$

$$V_A + V_B = -V_o$$



~~$\frac{V_A - V_t}{2R}$~~

$$\frac{V_A}{R} = i_1 =$$

$$\frac{V_A - V_-}{R} = \frac{V_- - V_o}{R} = i_1$$

$$\frac{V_B - V_t}{R} = \frac{V_t - 0}{R} = i_2$$

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$$V_B = 2V_F$$

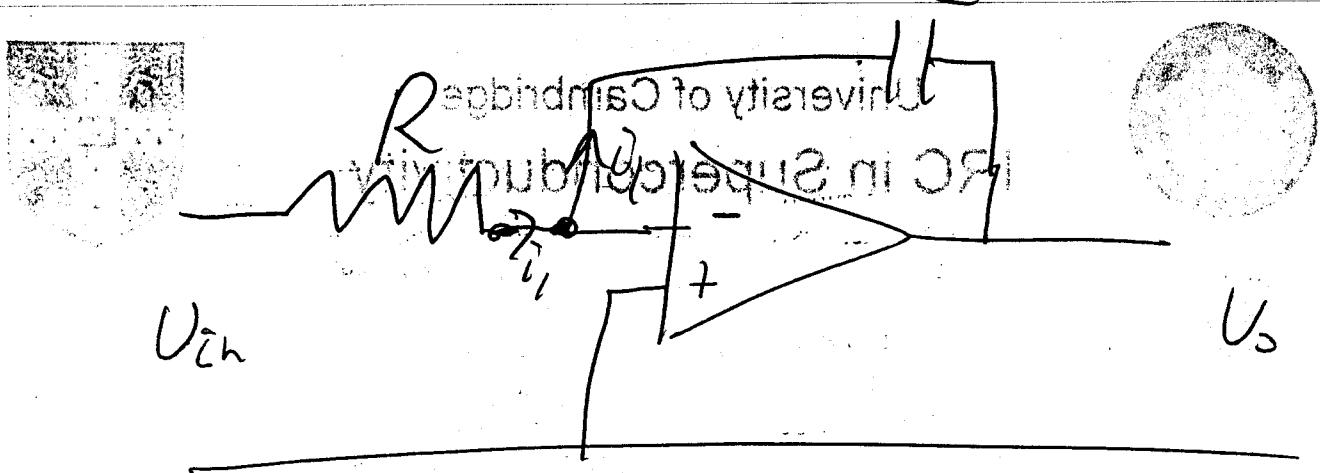
$$V_F = \frac{V_B}{2}$$

$$V_F = V_-$$

$$V_A - \frac{V_B}{2} = \frac{V_B}{2} - V_b$$

$$V_A - V_B = V_b$$

$$V_b = V_B - V_A$$



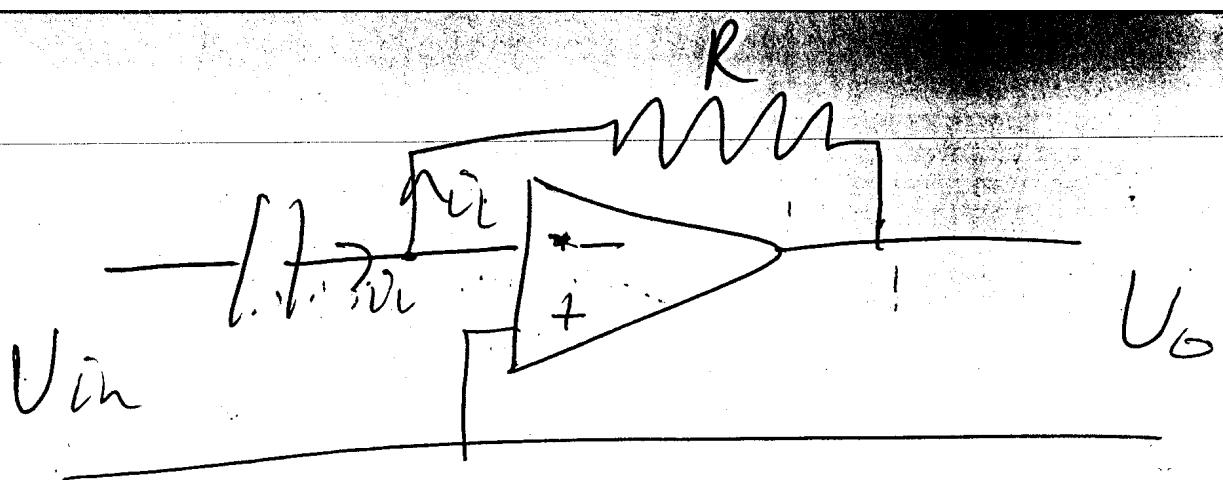
$$\frac{U_{in}}{R} = i_1$$

$$i_1 = \frac{C d(V_- - V_o)}{dt}$$

$$= - \frac{CdV_o}{dt}$$

$$\frac{dV_o}{dt} = - \frac{1}{RC} U_{in}$$

$$V_o = - \frac{1}{RC} \int U_{in} dt + \text{const}$$

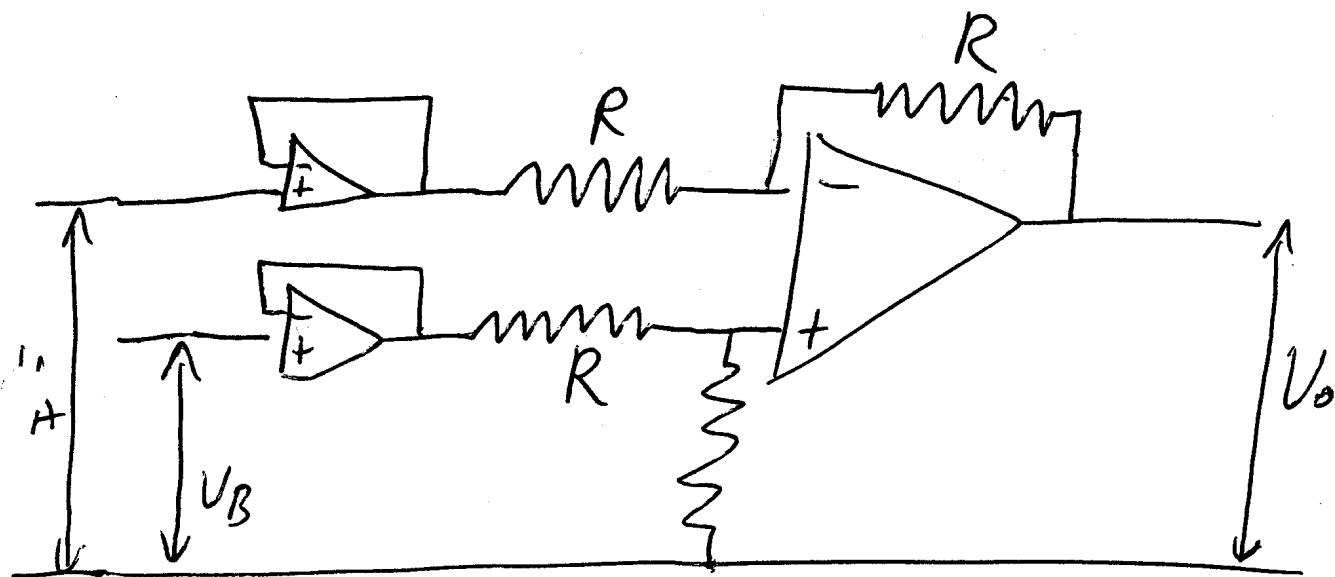


$$C \frac{d}{dt} \frac{U_{in} - U_-}{R} = i_1 = \frac{U_- - U_o}{R}$$

$$U_o = -RC \frac{dU_{in}}{dt}$$

O. High output impedance transducers.

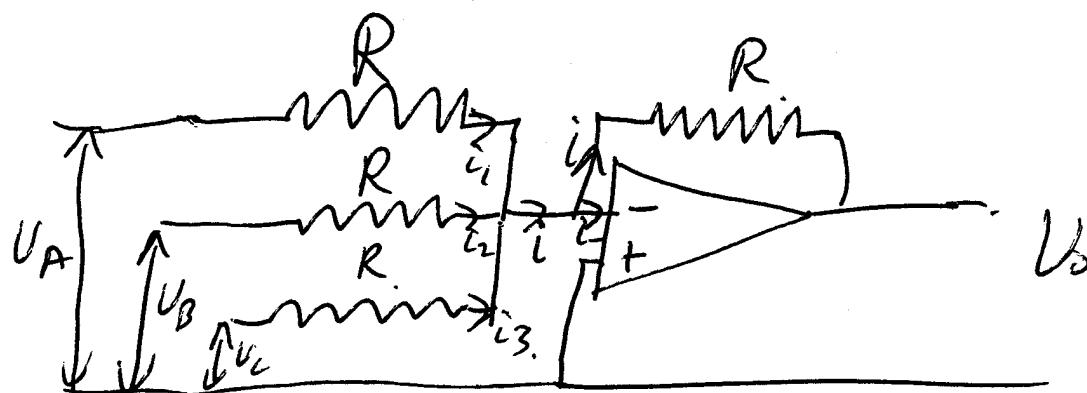
cannot provide much current, so need to use follower circuits to buffer them.



The followers have low output impedance & provide enough current for the next circuit.

② Transducer has low output impedance, so can provide lots of current.

To add 3 voltages together try :-



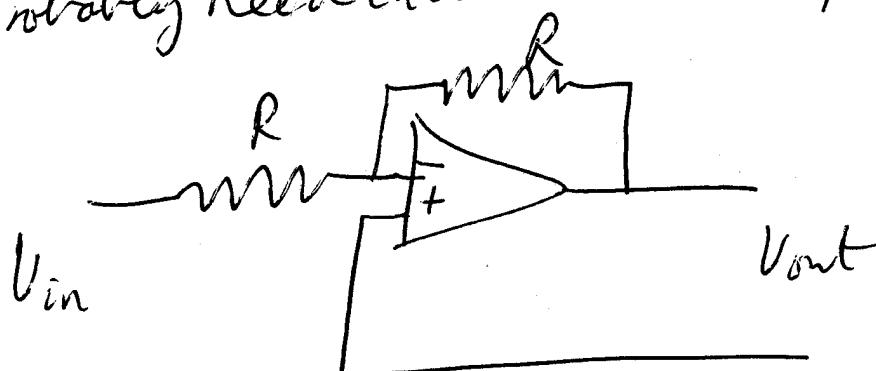
$$i_- = 0 \quad V_- = V_+ = 0$$

$$\text{So } i_1 + i_2 + i_3 = i$$

$$\frac{V_A}{R} + \frac{V_B}{R} + \frac{V_C}{R} = i = -\frac{V_o}{R}$$

$$\text{So } V_A + V_B + V_C = -V_o$$

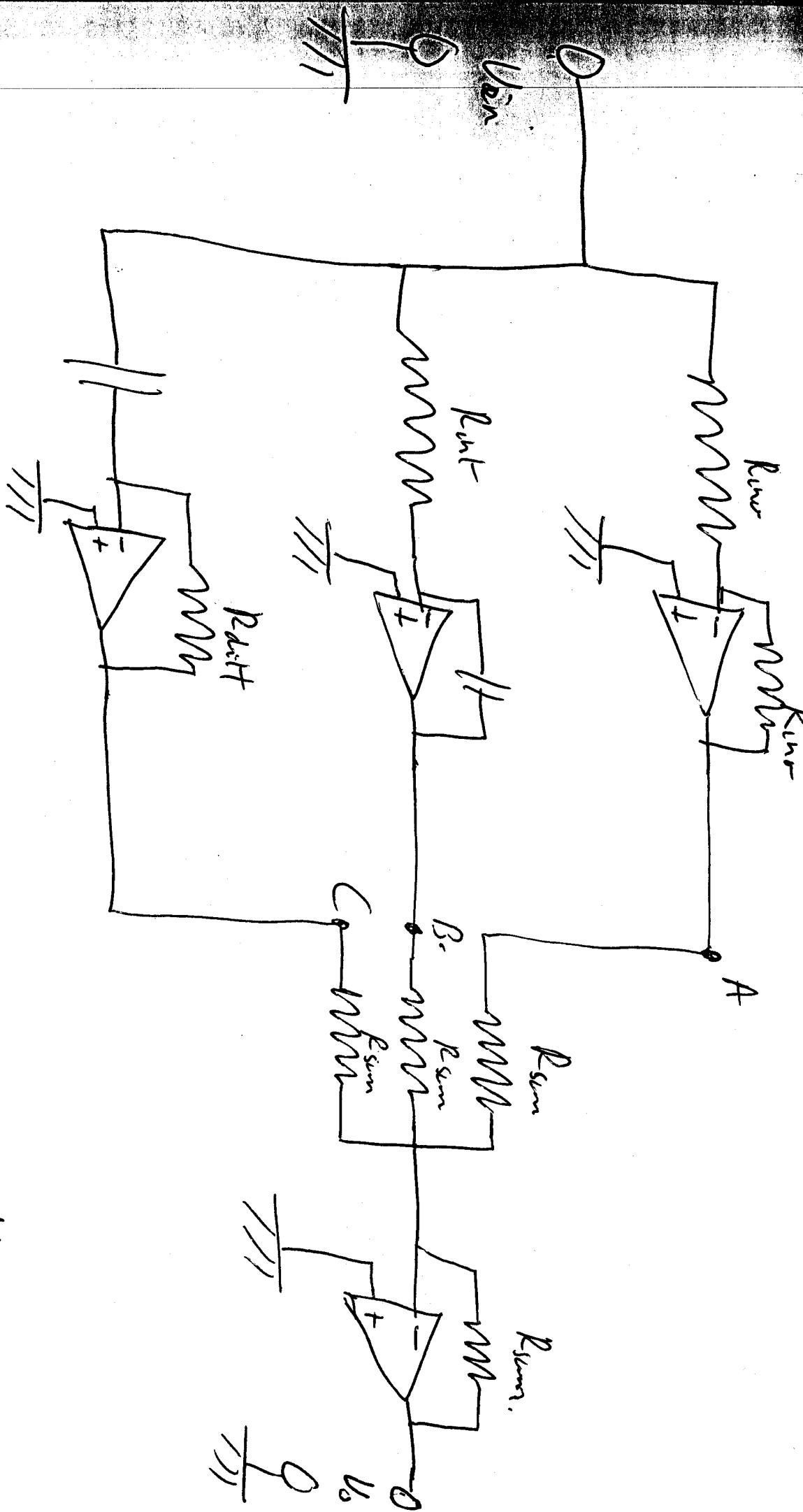
So probably need inverter on output



$$V_{\text{out}} = -V_{\text{in}}$$

No integrator + diff produce $-\frac{dV}{dT} \propto -\int V dt$ so use capacitor

So over all unit is:-



$$V_A = -V_{in} \quad V_B = -\frac{1}{RC}V_{in}t + C - \quad V_C = -\frac{RL}{R+L} \frac{dV_{in}}{dt}$$

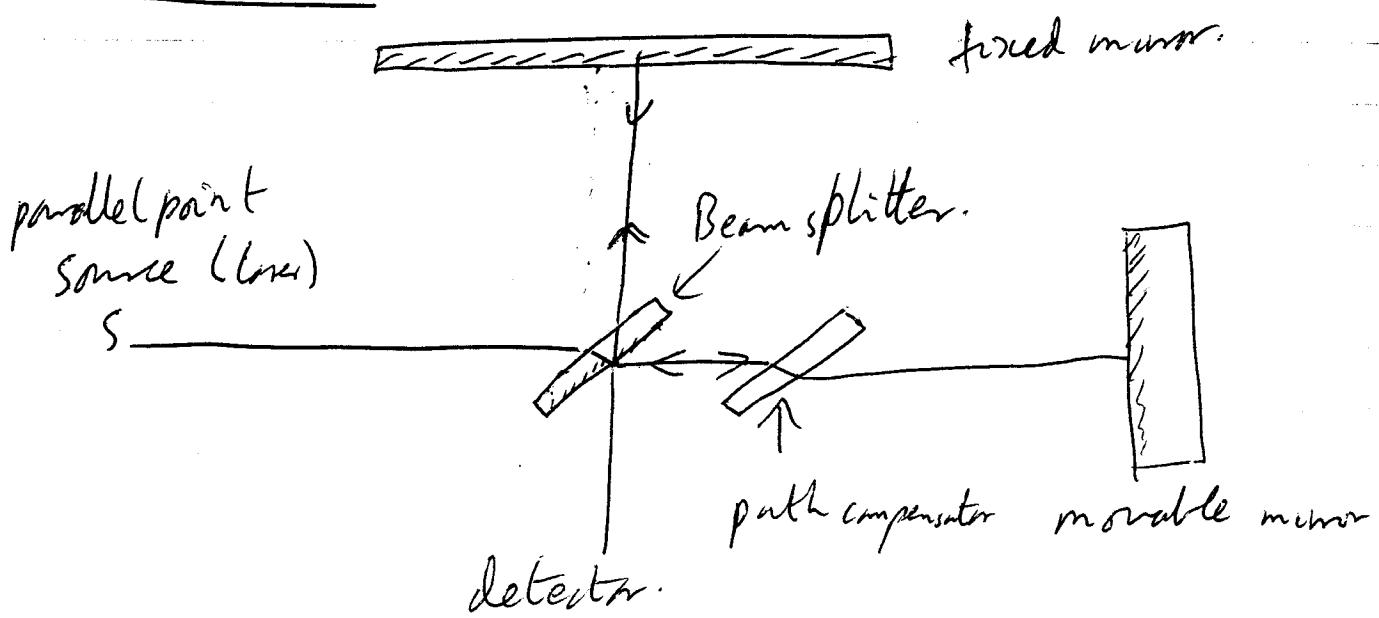
$$V_O = -(V_A + V_B + V_C)$$

⑤ I think this question is asking for descriptions of the Michelson interferometer and the Fabry-Pérot interferometer.

Micelson = 2 beam

Fabry Perot = Multiple beam.

Micelson



If phase difference is δ , total amplitude at detector from interference of the two beams is

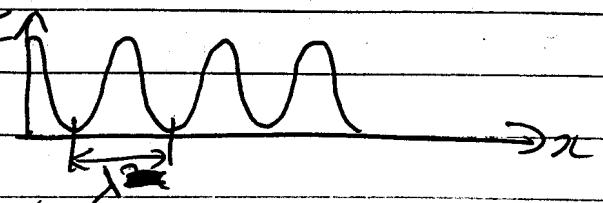
$$4 \propto 1 + \exp i\delta$$

hence intensity $14I^2 \propto (1 + \exp i\delta)(1 + \exp -i\delta)$
 $\propto 2(1 + \cos \delta)$
 $\propto \cos^2 \frac{\delta}{2}$

$$\delta = \frac{2\pi x}{\lambda} \quad x = \text{path difference}$$

Hence as x is varied intensity varies.

For single wavelength



But for multiple wavelengths i.e. two wavelengths have two independent fringe systems. The sources have no reason to be coherent, so there is no interference & we have to add intensities. The two fringe systems will coincide for $x=0$. If the different wavelengths will make the fringes go out of phase for some value of x & the maxima of one system will coincide with minima of the other.

By measuring the path difference over which the visibility of the fringes goes through a period we can determine wavelength difference $\Delta\lambda$ because

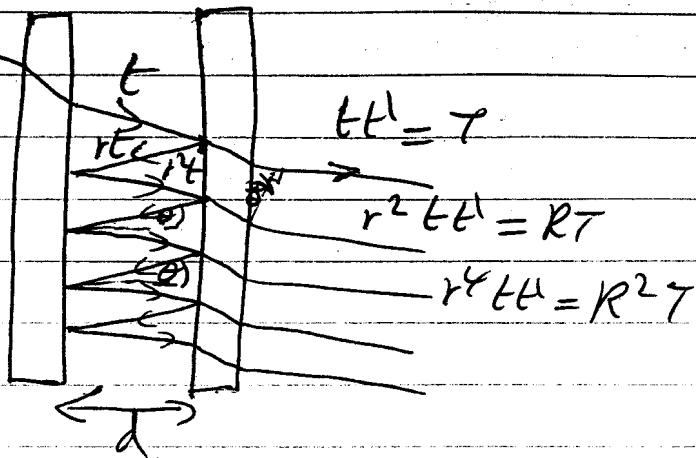
if $x = (N+1)\lambda = N(\lambda + \Delta\lambda)$ for one period of visibility

we have $x = \left(\frac{x}{\lambda + \Delta\lambda} + 1\right)\lambda$

$$\Rightarrow x\cancel{\lambda} + x\Delta\lambda = \cancel{x\lambda} + \lambda^2 + \lambda\Delta\lambda$$

$$\Rightarrow \Delta\lambda = \frac{\lambda^2}{x-\lambda} \sim \frac{\lambda^2}{x}$$

Fabry Perot.



Two half silvered mirrors a distance d apart.

First beam transmitted twice is amplitude = $t't$

Second beam reflected twice, transmitted twice & has extra path length.

$$x = 2d \cos \theta \quad (\text{from geometry})$$

Hence extra phase difference $\delta = \frac{4dx \cos \theta}{\lambda}$

The total amplitude is then.

$$A = T(1 + Re^{i\delta} + R^2 e^{2i\delta} + R^3 e^{3i\delta})$$

$$= \frac{T}{1 - Re^{i\delta}}$$

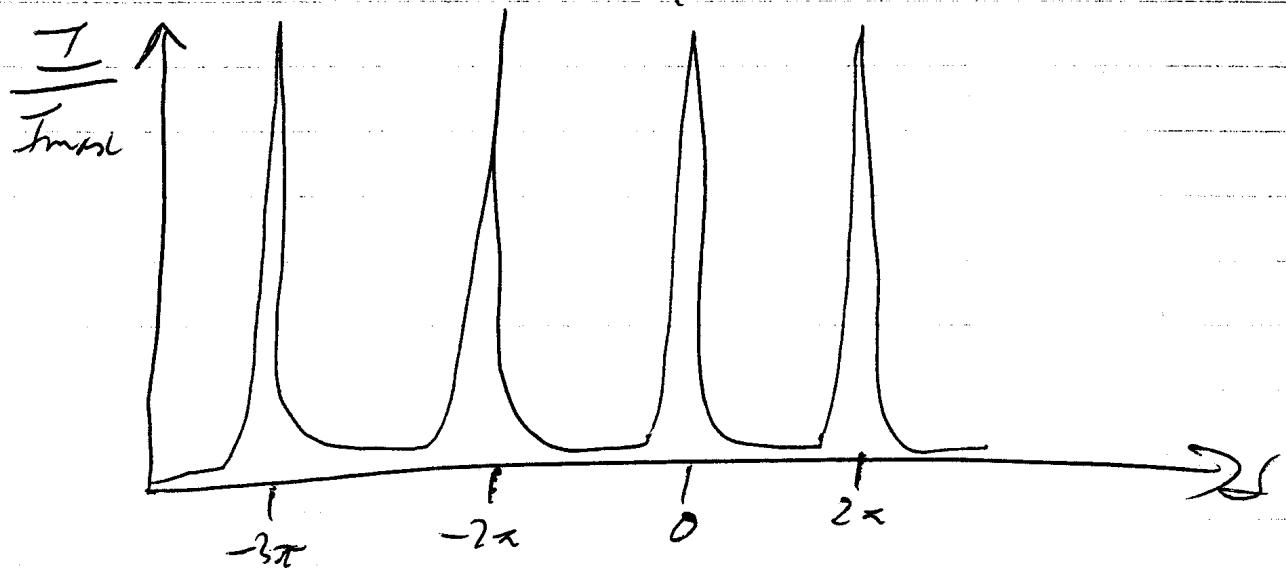
The intensity transmission is thus.

$$|A|^2 = \frac{T^2}{(1 - R \cos \delta)^2 + R^2 \sin^2 \delta} = \frac{T^2}{(1 - R)^2} \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2 \frac{\delta}{2}}$$

This is a very sharp function of δ
 which gives rise to very clear & distinct lines
 for each wavelength to enter the spectrometer.

The resolving power can be adjusted by changing the

Finesse $\gamma = \frac{\pi R^{1/2}}{1-R}$.



The sharpness of the peaks obtained with the multilens method, gives it a clear advantage over the two lens method.

Could also mention Fourier transform spectroscopy for Michelson case

Advantages, Fabry Perot is compact as it requires the two 'mirrors' of the etalon. Michelson larger.

Have already obtained formula probably
should have gone into more detail.

Resolving power:

Half width of peak is defined as change of
needed to reduce intensity to half maximum value

$$\frac{I_{\text{out}}}{I_n} = \frac{\tau^2}{(1-R)^2} \cdot \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2 \frac{\delta}{2}}$$

$$\text{If } \frac{I_{\text{out}}}{I_n} = \frac{1}{2} \cdot \frac{\tau^2}{(1-R)^2}$$

$$\frac{4R}{(1-R)^2} \sin^2 \frac{\delta_{1/2}}{2} = 1$$

$$\text{if } \delta_{1/2} = \text{small} \quad \frac{4R}{(1-R)^2} \left(\frac{\delta_{1/2}}{2}\right)^2 = 1$$

$$\Rightarrow \delta_{1/2} = \frac{1-R}{R^{1/2}}$$

\mathcal{F} = Finesse is defined as ratio of peak spacing, $\pi\delta = 2\pi b$ to
full width at half maximum

$$\mathcal{F} = \frac{2z}{2\delta_{1/2}} = \frac{\pi R^{1/2}}{1-R}$$

If ~~λ~~ is value of change in λ between two peaks reported to separate them in δ by full width half maximum

$$\Delta \lambda = 2\delta_{1/2}$$

then since $\delta = \frac{4\pi d \cos \theta}{\lambda}$

$$2\delta_{1/2} = d\delta = - \frac{4\pi d \cos \theta \, d\lambda}{\lambda^2}$$

$$\begin{aligned} \text{hence } \frac{\lambda}{d\lambda} &= \frac{4\pi d \cos \theta}{2\delta_{1/2} \lambda} \\ &= \frac{2\pi d \cos \theta}{\lambda \delta_{1/2}} \end{aligned}$$

$$\text{But } \frac{\lambda}{d\lambda} = \frac{2x}{2\delta_{1/2}} = \frac{\pi}{\delta_{1/2}}$$

$$\text{hence } \frac{\lambda}{d\lambda} = \frac{2d \cos \theta}{\lambda} \frac{x}{2\delta_{1/2}}$$

At a maximum of intensity ~~δ~~ $\delta = \frac{\text{integral number}}{2\pi}$

$$\text{so } 2\pi \frac{2d \cos \theta}{\lambda} = 2\pi m.$$

\Rightarrow

$$\frac{\lambda}{d\lambda} = n \frac{x}{2\delta_{1/2}} \quad m = \text{integer.}$$

$$\lambda = 585 \text{ nm} \quad \Delta\lambda = 10^3 \text{ nm.}$$

$$R = 0.95$$

$$\text{Hence } \gamma = \frac{\sim \gamma}{1 - 0.95} \\ = 61.2$$

$$\frac{\lambda}{d\lambda} = \frac{585}{10^{-3}} = 585000$$

$$\text{so } \frac{\lambda}{d\lambda} = \frac{2d \cos \theta}{\lambda} \quad \gamma$$

minimum value of d is when $\cos \theta = 1$

$$\text{so } d = \frac{585000 \times 585 \times 10^{-9}}{61.2 \times 2} \\ = 2.42 \text{ mm.}$$