

**2001**

*Paper 1*

2001 I

1  $E = 5 \times 10^{-2} \text{ Vm}^{-1}$  rms

$P = 1 \text{ MW}$  isotropic

$|N| = P/2\pi r^2 = E^2/2\sigma$

$$r = \sqrt{\frac{P Z_0}{4\pi E^2}}$$

$$\underline{r = 1.1 \times 10^5 \text{ m}}$$

2  $L_A$  and  $L_B$  are inductances of A and B

$\frac{1}{4}$  of flux due to current in A links B

$$\phi_2 = L_2 I_2 + M I_1$$

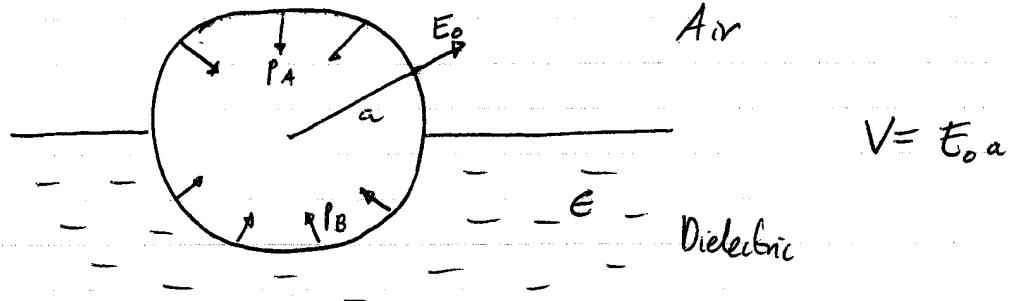
$$\phi_1 = L_1 I_1 + M I_2$$

$$I_2 = 0 \Rightarrow \phi_1 = L_1 I_1, \quad \phi_2 = M I_1, \quad \underline{\phi_2/\phi_1 = \frac{1}{4} = \frac{M I_1}{L_1 I_1} = \frac{M}{L_1}}$$

$$I_1 = 0 \Rightarrow \phi_2 = L_2 I_2, \quad \phi_1 = M I_2, \quad \underline{\phi_1/\phi_2 = \frac{M I_2}{L_2 I_2} = \frac{M}{L_2}}$$

$$\underline{\underline{\phi_1/\phi_2 = \frac{L_1}{4 L_2}}}$$

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$$V = \epsilon_0 a$$

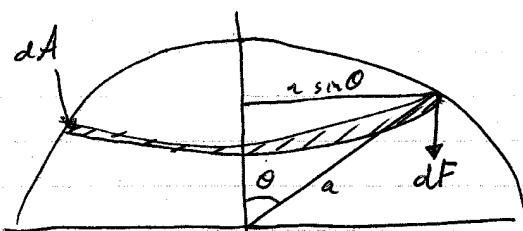
$$U_{\text{conductor}} = k D \cdot E = 0 \quad \therefore E_{\text{conductor}} = 0$$

$$U_{\text{Air}} = k D \cdot E = k \epsilon_0 E_0^2 = p_A$$

$$U_{\text{Dielectric}} = k D \cdot E = k \epsilon_0 \epsilon E_0^2 = p_B$$

All forces in any directions other than  $z$  dim cancel.

Subtract  $p_A$  from  $p_B$  and consider ~~top~~<sup>bottom</sup> hemisphere only



Consider small area  $dA$ .

$$dA = 2\pi a \sin\theta a d\theta$$

$$\text{Pressure on } dA \text{ is } p_B - p_A \quad \therefore \text{Force } \propto (p_B - p_A) dA = k \epsilon_0 (\epsilon - 1) \frac{V^2}{a^2} dA$$

But only want component  $dF$  in  $z$  dim.

$$\therefore dF = k \epsilon_0 (\epsilon - 1) \frac{V^2}{a^2} \cos\theta dA$$

$$F = \int_0^{2\pi} k \epsilon_0 (\epsilon - 1) \frac{V^2}{a^2} \cos\theta 2\pi a^2 \sin\theta d\theta$$

$$= k \epsilon_0 (\epsilon - 1)$$

9 (i) Bookwork.

2001 9 (ii)



$$\rightarrow H_1$$

$$\rightarrow H_m$$

$$B_z = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}}$$

$$\text{B.C. : } H_1 \text{ continuous} \Rightarrow H_m = H_2$$

$$\Rightarrow \frac{B_1}{\mu_r \mu_0} = \frac{B_2}{\mu_0}$$

$$\Rightarrow B_m = \mu_r B_z = \frac{(H+M)}{\mu_0}$$

$$\Rightarrow M = \frac{\mu-1}{\mu_0} B_z$$

$$m = V B_z \frac{(\mu-1)}{\mu_0} \quad \circ z \text{ down}$$

$$F = (\underline{m} \cdot \nabla) \underline{B} = V B_z \frac{(\mu-1)}{\mu_0} \frac{\partial}{\partial z} \underline{B}$$

$$\frac{\partial \underline{B}}{\partial z} = (0, 0, \frac{\mu_0 I a^2 2z (-\frac{3}{2})}{2(a^2 + z^2)^{5/2}})$$

$$F = \frac{\mu_0 (\mu-1) V I^2 a^4 z^3}{4(a^2 + z^2)^4} \quad \text{Force is attractive for } \mu > 1$$

If shape changed then B.C. will give a different M

# NATSCI: Part 1B, Part II (General)

Tues. 29 May 2001.

## Advanced Physics (I)

D12

Convolution thm.

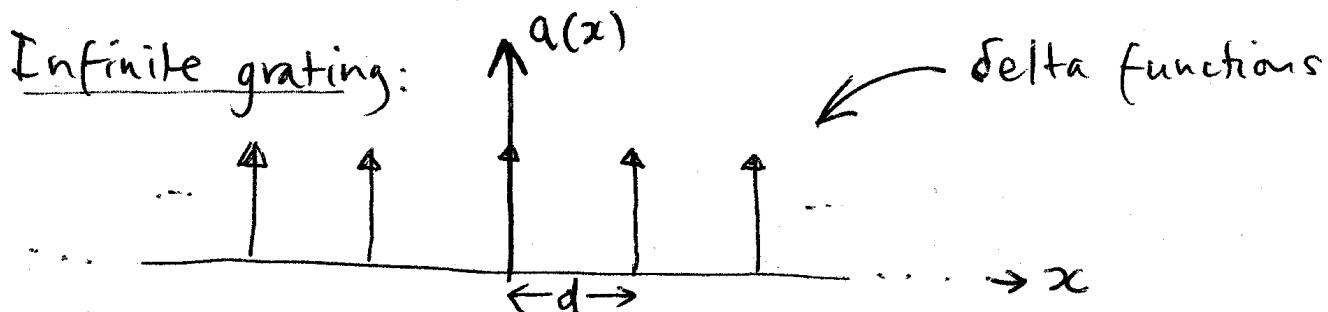
$$(i) \text{FT}[f * g] = \text{FT}[f] \times \text{FT}[g]$$

$$(ii) \text{FT}[f \times g] = \text{FT}[f] * \text{FT}[g]$$

FT = Fourier Transform.

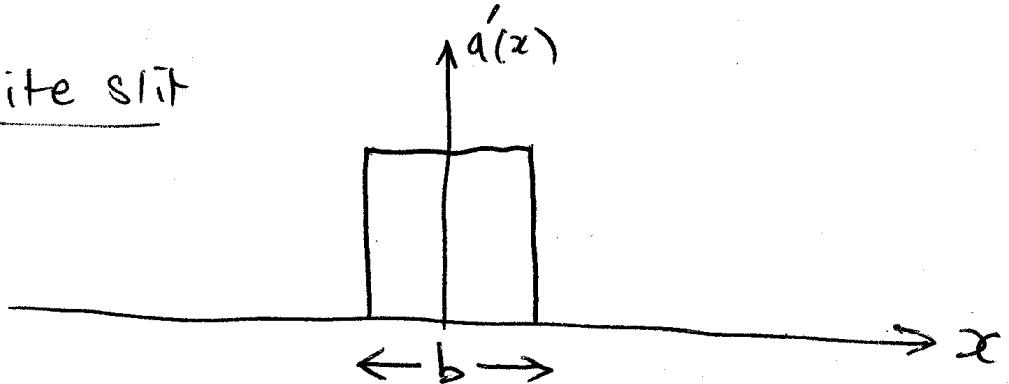
\* = Convolution

X = multiplication

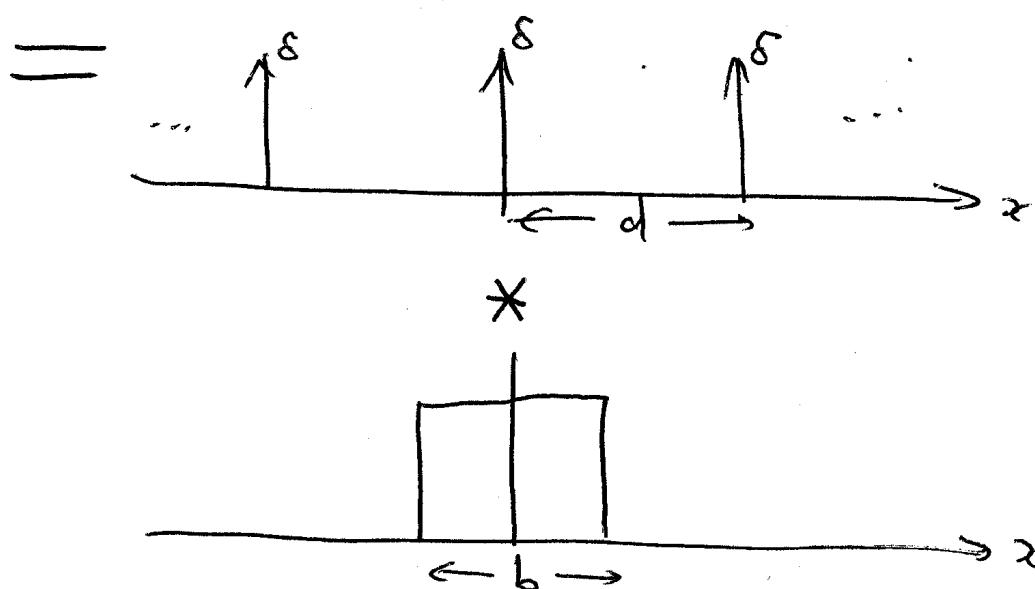
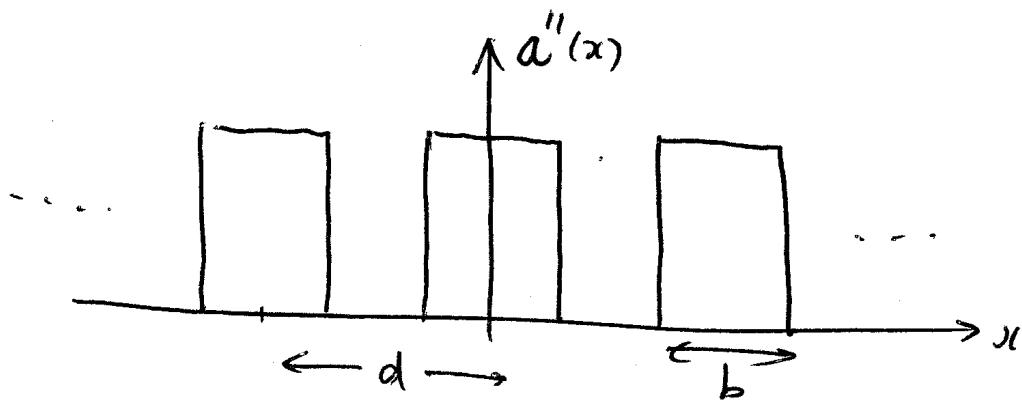


$a(x)$  = amplitude function

Finite slit



Regular array of finite slits:



$$= a(x) * a'(x).$$

$$\therefore \text{FT}[a''(x)] = \text{FT}[a(x)] \times \text{FT}[a'(x)]$$

$$\propto \delta\left(q - \frac{2\pi n}{d}\right) \times \text{sinc}\left(\frac{b}{2}q\right)$$

Where  $q = \frac{2\pi}{\lambda} \sin \theta$   $\nwarrow$  observation angle.

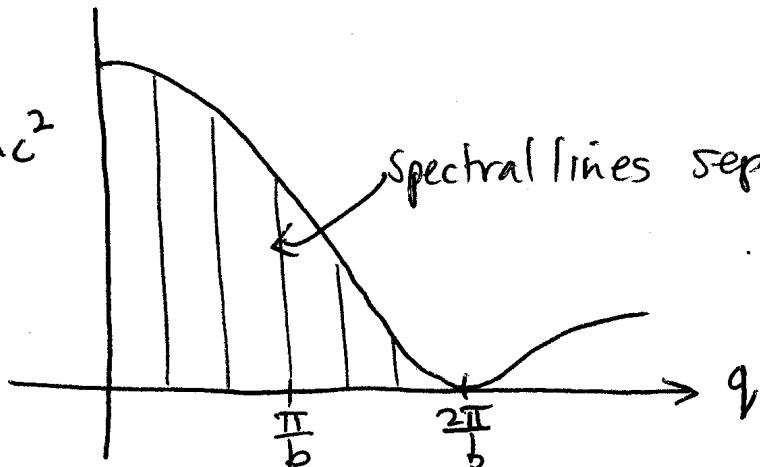
Grating: 50 slits / mm.  $\Rightarrow d = 2 \times 10^{-5} \text{ m}$

$$\lambda = 450 \text{ nm}$$

$$b = \frac{1}{3} d = \frac{2}{3} \times 10^{-5} \text{ m}$$

Envelope Intensity

$\propto \text{sinc}^2$



Spectral lines separated by  $\frac{2\pi}{d}$ , occur

$$\text{at } q_l = \frac{2\pi l}{d}, l=0, 1, 2, \dots$$

(a)  $\text{sinc}^2 \rightarrow 0$  when  $q = \frac{2\pi n}{b}$   $n=1, 2, \dots$

$n=1 : q = \frac{2\pi}{b} = \frac{b\pi}{d}$ . Coincides with  $q_3$   $\Rightarrow$  missing peak.

In fact every 3rd spectral line is missing.

(b) 1st:  $q_1 = \frac{2\pi}{d} \Rightarrow \text{sinc}^2\left(\frac{\pi}{3}\right)$

2nd:  $q_2 = \frac{4\pi}{d} \Rightarrow \text{sinc}^2\left(\frac{2\pi}{3}\right)$

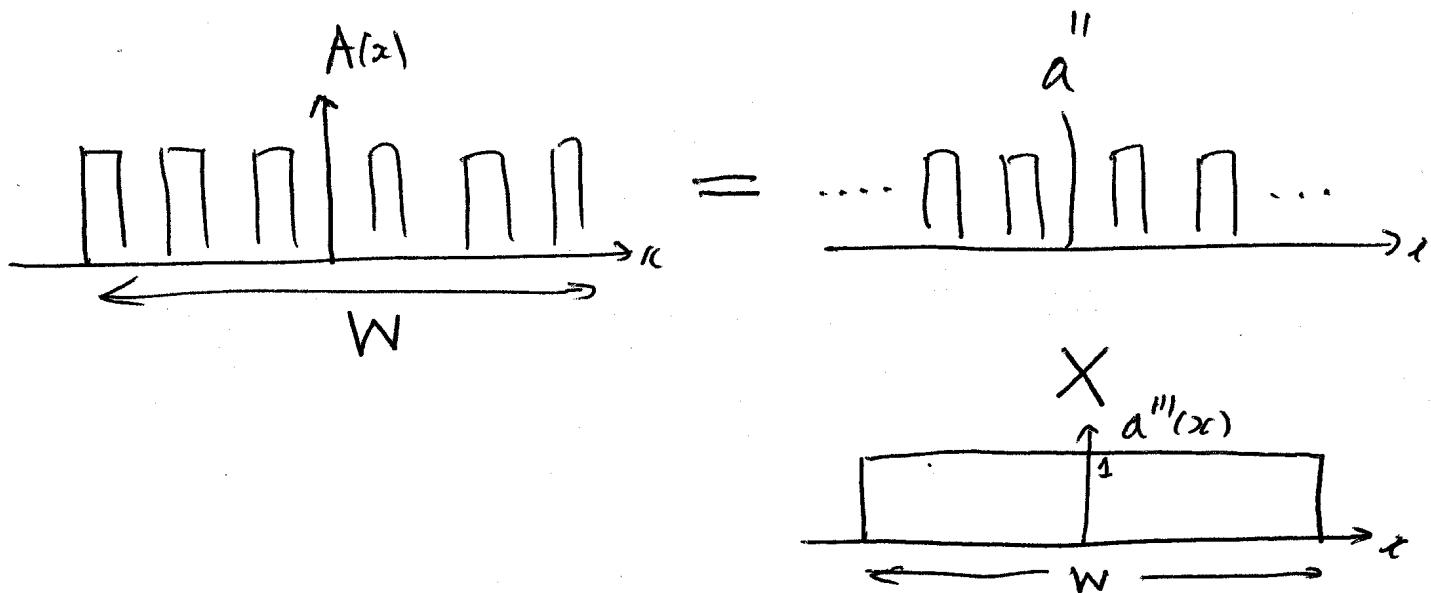
0<sup>th</sup>: 1

$\left. \begin{array}{l} l \quad 0; 1; 2 \\ \text{ratio} \quad 1:0.68:0.17 \\ \text{of} \\ \text{Intensity} \end{array} \right\}$

Angle of 0;  $\frac{\lambda}{d}$ ;  $\frac{2\lambda}{d}$   
emergence

(small angles)

(C) Finite grating, width  $W$ :

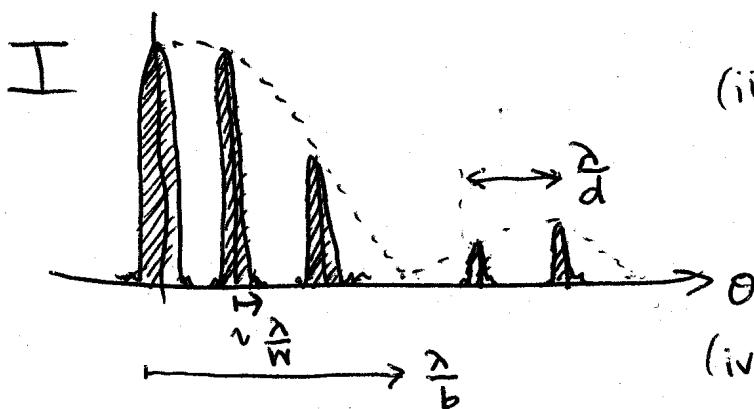


$$\therefore \text{FT}[A(x)] = \text{FT}[a''(x)] * \text{FT}[a'''(x)]$$

$$\propto \delta(q - \frac{2\pi n}{d}) \text{sinc}\left(\frac{b}{2}q\right) * \underbrace{\text{sinc}\left(\frac{W}{2}q\right)}$$

Since  $W \gg b$ , this is a small feature of the diffraction pattern  $\rightarrow$  A narrow sinc.

The convolution decorates each peak with a sinc profile.



- (i)  $W \rightarrow \infty \Rightarrow$  as before, infinitely thin peaks
- (ii)  $W \rightarrow 0 \Rightarrow$  Peaks broaden until fills envelope, leaving a  $\text{sinc}^2$  pattern.
- (iv) As  $W$  increases, more light passes  $\Rightarrow$  brighter.