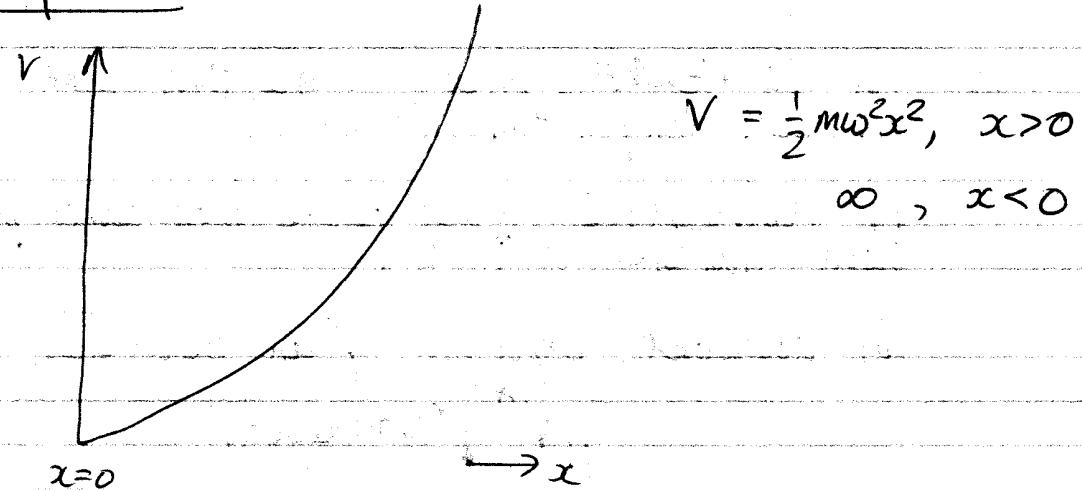


2003

Paper 2

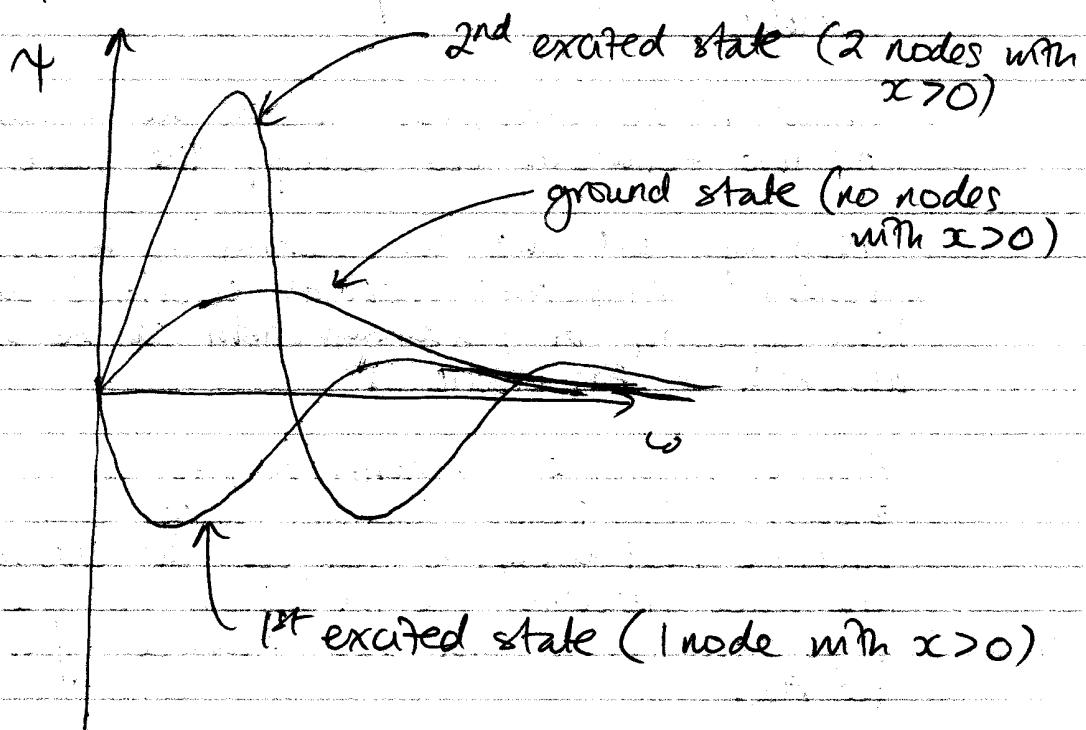
2003 Paper 2

~ A1.



Since $V = \infty$ at $x = 0$, all wavefunctions must go to zero at $x = 0$.

The positive- x potential is just that of a harmonic oscillator, so the allowed wavefunctions of this potential are just the allowed wavefunctions of the harmonic oscillator that pass through zero at $x = 0$, i.e. the $n=1, 3, 5$ etc. states.



Since $E_n = (n + \frac{1}{2}) \hbar \omega$ for the full harmonic oscillator, the energies of these states are

$$E = \frac{3}{2} \hbar \omega, \frac{7}{2} \hbar \omega, \frac{11}{2} \hbar \omega.$$

A2. An ISOTHERMAL process is one that takes place at a constant temperature ($T = \text{const.}$).

An ADIABATIC process is one that takes place with no heat transfer ($dQ = 0$).

For adiabatic expansion of an ideal gas,

$$pV^\gamma = \text{constant},$$

where $\gamma = \frac{C_p}{C_v} = \frac{\text{heat capacity at constant pressure}}{\text{heat capacity at constant volume}}$.

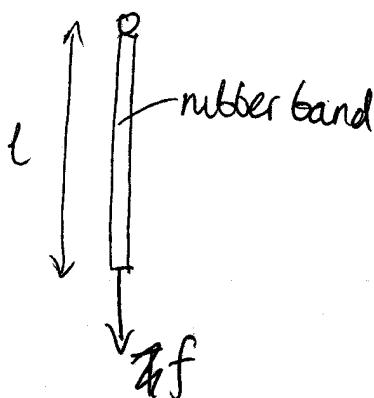
A3. A fermion is a particle with half-integer (i.e. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ etc.) intrinsic angular momentum (or spin). The wavefunction of a two-fermion system is antisymmetric under particle exchange. Examples of fermions: protons, neutrons, electrons, Helium-3 nucleus.

Fermi-Dirac statistics include the exclusion principle - that no two fermions can occupy the same quantum state.

Very important when considering the behaviour of electrons in metals - properties such as electronic contributions to the heat capacity, or paramagnetism can be explained by assuming as FD statistics.

Another example: "degeneracy pressure" in white dwarf stars. Dead stars contract under gravity. But this cannot continue indefinitely, as then all the electrons would be in one place violating the exclusion principle. The electrons repel, causing a force that balances gravity, and holds up the star.

A4.



We assume a form for the tension f

$$f \approx AT(l - l_0)$$

where $A = \text{const}$, $T = \text{temperature}$,
 $l_0 = \text{unstretched length of band}$

Since tension f is constant, an increase in T must cause a decrease in $(l - l_0)$, i.e. the band will contract on heating.

The contraction is also favoured entropically, as the ~~rubber~~ polymer molecules are less ordered.

A5. Kinetic energy = 1 MeV is much less than rest mass energy of α -particles, so can use non-relativistic expression

$$KE = \frac{p^2}{2m_\alpha}$$

$$\text{Since } m_\alpha = 6.64 \times 10^{-27} \text{ kg, } KE = 1.6 \times 10^{-13} \text{ J}$$

$$p = 4.61 \times 10^{-20} \text{ kg m s}^{-1}$$

This is the typical momentum of an α -particle and we guess that $\Delta p \approx p$, and use

$$\Delta p \Delta x \gtrsim \frac{1}{2}\hbar$$

to see that

$$\Delta x \lesssim 1.1 \times 10^{-15} \text{ m}$$

i.e. less than the size of a nucleus, and ~~that~~ thus it is possible that the α -particle ~~was~~ was a constituent of the original nucleus.

B6. Impedance is an measure of the ~~energy~~

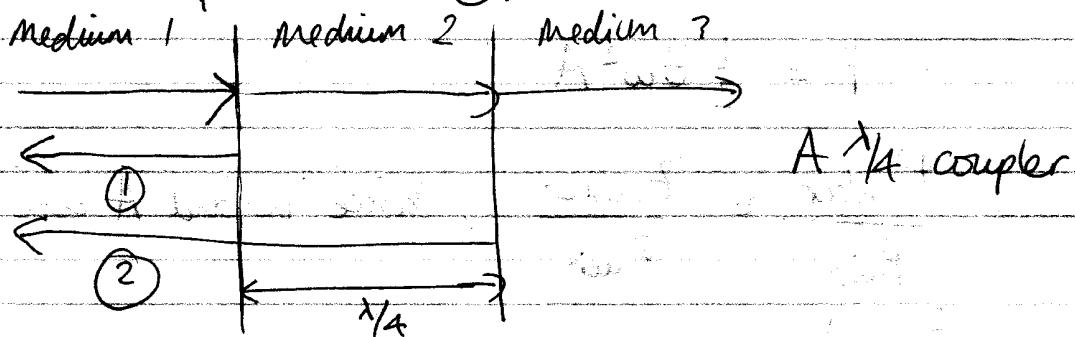
B6. Waves transport energy away from their source. The amplitude of a wave for a given force will depend on the properties of the medium through which the wave travels. The impedance is defined as

$$Z = \frac{\text{driving force}}{\text{amplitude response}}$$

Impedance matching refers to the idea of making two adjacent media have as close impedances as possible. This will minimise reflection at the boundary.

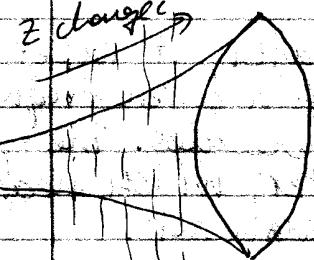
Two examples where it is important are: camera lenses, and musical instruments.

In camera lenses, an intervening layer is placed between the air and the lens, the impedance of which is chosen so that the reflected wave① interferes destructively with the re-reflected wave②.



Thus, most of the energy is transmitted, because the amount reflected from the medium 1-2 boundary is negligible.

Many musical instruments have a curved bell where the sound is emitted. This gradually opening pipe has a gradually changing impedance between the impedance of the air inside the clarinet, and the free air.



A horn

This gradual change means that there are infinitesimal ~~and~~ reflected waves created by infinitesimal changes in impedance. These will randomly differing phases that will have produce a very small total wave. Thus, most of the energy will be transmitted

~~Power & force of transverse velocity~~

$$P = \frac{1}{2} \operatorname{Re}[Fu^*], F = \text{force}, u = \text{transverse velocity}.$$

But, from the definition of impedance, $F = zu$, where z is impedance

$$P = \frac{1}{2} \operatorname{Re}[zuu^*]$$

$$= \frac{1}{2} \operatorname{Re}[z] |u|^2$$

$$\text{If } \Psi = Ae^{i(\omega t - kx)}$$

$$u = \frac{d\Psi}{dt} = i\omega Ae^{i(\omega t - kx)}$$

and z is real,

$$\underline{P = \frac{1}{2} zw^2 A}$$

$$P = \frac{1}{2} zw^2 A$$

$$\frac{P_{\text{water}}}{P_{\text{air}}} = \frac{z_{\text{water}}}{z_{\text{air}}}, \text{ since } w \text{ and } A \text{ are the same}$$

$$z = \rho V$$

$$\frac{P_w}{P_A} = \frac{\frac{P_w V_w}{P_A V_A}}{\frac{(1000 \text{ kg m}^{-3})(1500 \text{ ms}^{-1})}{(1.2 \text{ kg m}^{-3})(340 \text{ ms}^{-1})}} = \underline{3700}$$

Because air is much less dense than water, it takes much less energy to generate a wave in air of a given frequency, simply because you are moving fewer molecules. The speed of sound in air is also lower than in water, which means it will take less energy to drive a wave through air.

The impedance change is less for F . Since impedance is proportional to F , and the impedance of water is larger than that of air, the force on the film in water will be higher. This will subject it to greater forces, and make it more likely to suffer mechanical failure.

$$\sim \text{B10. } p = |\psi|^2 = \psi^* \psi$$

$$\Rightarrow \frac{\partial p}{\partial t} = \psi \frac{\partial \psi^*}{\partial t} + \psi^* \frac{\partial \psi}{\partial t}$$

Schrödinger:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

and the conjugate:

$$-i\hbar \frac{\partial \psi^*}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi^*}{\partial x^2} + V\psi^*$$

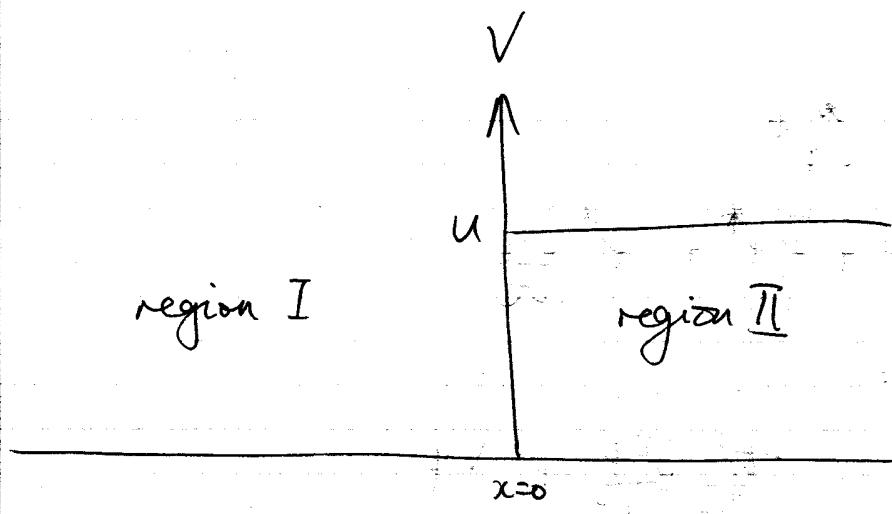
hence

$$\begin{aligned} \frac{\partial p}{\partial t} &= \psi \left\{ -i\hbar \frac{\partial^2 \psi^*}{2m \partial x^2} + \frac{iV}{\hbar} \psi^* \right\} \\ &\quad + \psi^* \left\{ i\hbar \frac{\partial^2 \psi}{2m \partial x^2} - \frac{iV}{\hbar} \psi \right\} \\ \frac{\partial p}{\partial t} &= \frac{i\hbar}{2m} \left(\psi^* \frac{\partial^2 \psi}{\partial x^2} - \psi \frac{\partial^2 \psi^*}{\partial x^2} \right) \end{aligned}$$

We know also that

$$j = \frac{\hbar}{2im} \left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x} \right)$$

$$\begin{aligned} \frac{\partial j}{\partial x} &= \frac{\hbar}{2im} \left(\psi^* \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial \psi^* \partial \psi}{\partial x \partial x} - \psi \frac{\partial^2 \psi^*}{\partial x^2} - \frac{\partial \psi \partial \psi^*}{\partial x \partial x} \right) \\ &= \frac{\hbar}{2im} \left(\psi^* \frac{\partial^2 \psi}{\partial x^2} - \psi \frac{\partial^2 \psi^*}{\partial x^2} \right) \\ &= -\frac{i\hbar}{2m} \left(\psi^* \frac{\partial^2 \psi}{\partial x^2} - \psi \frac{\partial^2 \psi^*}{\partial x^2} \right) \\ &= -\frac{\partial p}{\partial t}, \text{ as req'd.} \end{aligned}$$



$$\Psi_I = e^{ik_1 x} + r e^{-ik_1 x}, \quad k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\Psi_{II} = t e^{ik_2 x}, \quad k_2 = \sqrt{\frac{2m(E-U)}{\hbar^2}}$$

Ψ must be continuous at the boundary

$$\Rightarrow 1 + r = t \quad \textcircled{1}$$

$\frac{\partial \Psi}{\partial x}$ must be continuous at the boundary

$$\Rightarrow k_1 - k_1 r = k_2 t \quad \textcircled{2}$$

$$\text{from } \textcircled{1} \quad k_1 + k_1 r = k_1 t$$

$$\Rightarrow 2k_1 = (k_1 + k_2) t$$

$$\Rightarrow 2k_1 r = (k_1 - k_2) t$$

$$2k_1 r = (k_1 - k_2) / 2k_1$$

$$r = \frac{(k_1 - k_2)}{(k_1 + k_2)}$$

$$r = \frac{(k_1 - k_2)}{(k_1 + k_2)}$$

$$r = \frac{\sqrt{\frac{2mE}{\hbar^2}} - \sqrt{\frac{2m(E-U)}{\hbar^2}}}{\sqrt{\frac{2mE}{\hbar^2}} + \sqrt{\frac{2m(E-U)}{\hbar^2}}}$$

$$r = \frac{\sqrt{E} - \sqrt{E-U}}{\sqrt{E} + \sqrt{E-U}}$$

$$t = \frac{2\sqrt{\frac{2mE}{k^2}}}{\sqrt{\frac{2mE}{k^2}} + \sqrt{\frac{2m(E-U)}{k^2}}}$$

$$t = \frac{2\sqrt{E}}{\sqrt{E} + \sqrt{E-U}}$$

$$|t|^2 = \frac{4E}{(\sqrt{E} + \sqrt{E-U})^2}$$

$$\frac{k_2}{k_1} = \sqrt{\frac{E-U}{E}}$$

$$|r|^2 = \frac{(\sqrt{E} - \sqrt{E-U})^2}{(\sqrt{E} + \sqrt{E-U})^2}$$

$$\begin{aligned} |r|^2 + \frac{k_2}{k_1} |t|^2 &= \frac{(\sqrt{E} - \sqrt{E-U})^2 + 4\sqrt{E(E-U)}}{(\sqrt{E} + \sqrt{E-U})^2} \\ &= \frac{E + E-U - 2\sqrt{E(E-U)} + 4\sqrt{E(E-U)}}{(\sqrt{E} + \sqrt{E-U})^2} \\ &= \frac{E + E-U + 2\sqrt{E(E-U)}}{(\sqrt{E} + \sqrt{E-U})^2} \\ &= \frac{(\sqrt{E} + \sqrt{E-U})^2}{(\sqrt{E} + \sqrt{E-U})^2} \\ &= 1, \text{ as req'd.} \end{aligned}$$

If $U > E$, the transmitted wave would be evanescent, and no energy would be transmitted into the barrier. The amplitude of the transmitted wave would decay exponentially.