2001/C13 by spacetime diagrams

Sanjoy Mahajan
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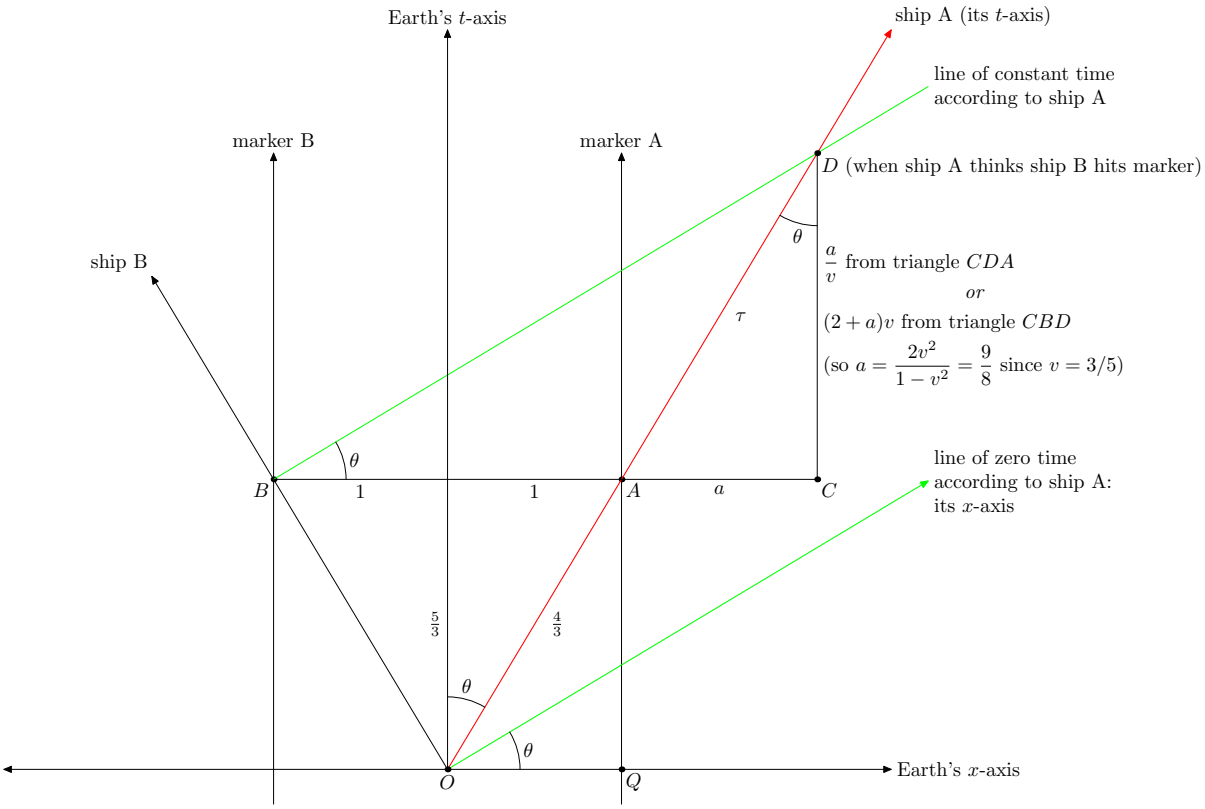
sanjoy@mrao.cam.ac.uk



In the first part of the question, you find that $AQ = 5/3$ and $v = 3/5$. When does ship A think it hits its marker? In other words, how long is OA? Lorentz geometry gives

$$OA^2 = \left(\frac{5}{3}\right)^2 - 1^2 = \left(\frac{4}{3}\right)^2,$$

so $OA = 4/3$.

When does ship B hits its marker, *according to ship A?* Ship B hits its marker at event B. From event B, draw ship A's line of constant time (green line). It intersects ship A's worldline – its time axis (red line) – at D. The interval OD is the time that ship A gives to event B. You can carefully draw this diagram, then measure OD with a ruler. Scale its length against the length of OA, which is 4/3. Try it!

Or use similar triangles. The length AC is an unknown, call it a. The time interval CD forms a side of two similar triangles. The resulting equation (see diagram) shows that $a = 9/8$. The triangle OQA is similar to ACD, so

$$\tau = \frac{4}{3}\,a = \frac{4}{3}\cdot\frac{9}{8} = \frac{3}{2}.$$

The proper time OD is then

$$\frac{4}{3} + \frac{3}{2} = \frac{17}{6}.$$

What did you find with the ruler?