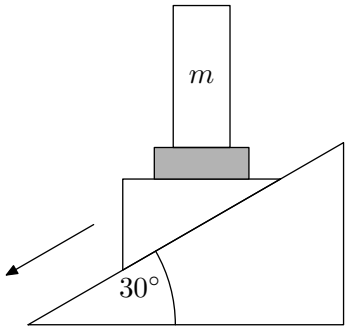
## Problem set 3

**1** *Tetrahedron*

In methane ($CH_4$), the four hydrogen atoms lie at the corners of a regular tetrahedron and the carbon atoms lies at the centre. Where is the centre of a tetrahedron with unit edge length? What is the angle between two C–H bonds? (Hint: Make an analogy, so that you can make an educated guess before doing fancy maths.)

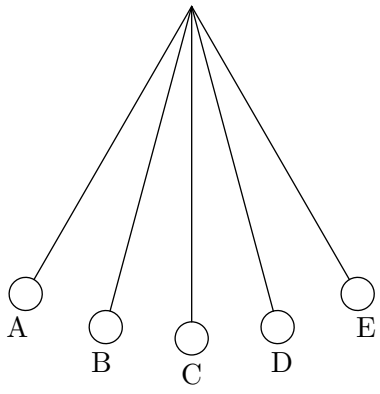
**2** *Skiing*

You (tall rectangle, with mass $m$) stand on a wedge sliding down a frictionless plane, as shown in the figure. What weight does the scale (shaded rectangle) read? Use clearly labeled, well-separated free-body diagrams.




**3** *Pendulum*

The figure shows a pendulum at five points in its swing; positions A and E are the extremes of the motion. On each bob, draw (approximately) the acceleration vector at that point in the swing. If the acceleration is zero (in which case there is no arrow to draw), simply circle the bob.




**4** *Twelve balls*

After all the discussions on the twelve-balls problem, now it's time to finish it! Here it is repeated from the vac problems:

You are given 12 balls, all of which are equal in weight except for one which is either heavier or lighter. You are also given a two-pan balance to use. In each use of the balance you may put

any number of the 12 balls on the left pan, and the same number on the right pan, and push a button to initiate the weighing. There are three possible outcomes: either the weights are equal, or the balls on the left are heavier, or the balls on the left are lighter. Your task is to design a strategy to determine which is the odd ball *and* whether it is heavier or lighter than the others *in as few uses of the balance as possible*. Can you prove that your strategy is optimal?

**5** *Interpreting equations*

Here you will study the well-known formula for the horizontal range of a rock. You throw a rock with velocity $v$ at an angle $\theta$ with respect to the ground. Its range is

$$R = \frac{2v^2}{g} \sin\theta \cos\theta. \tag{5.1}$$

You can increase your confidence in this result in a number of ways (parts a–e). (It may help for many of the parts to draw a diagram.)

a) Dimensional analysis: Check whether the dimensions are correct.

b) Consider limiting cases (for example, $\theta = 0$). Does the range formula work in these cases?

c) Give a physical argument for why the range contains a factor of $v^2$ (instead of, say, simply $v$ or $1/v$ or no $v$ at all). (Dimensional analysis, which you did in part a, is a mathematical argument; in this part, you are asked to reinforce the mathematics with a physical argument.)

d) Give a physical argument for the factor of 2.

e) Give a physical argument for the $1/g$ factor.

f) To derive (5.1), you have to neglect many effects (for example air resistance). List as many of these effects as you can. Let your imagination run; no effect is too small to mention here.