

Problem set 6

1 *Sun*

Question 27 from the Examples sheet.

2 *Colliders*

SLAC is the Stanford Linear Accelerator, which smashes electrons and positrons together at high speeds. Here you'll get to see why physicists build colliders, other than as a warmup for creating new ways to kill people (your tax dollars at work). I'll use $c = 1$ units to favor ideas over algebra.

a) As a crude model of a normal accelerator, imagine a collision between two particles of unit mass (imagine your favorite mass unit, for example, the mass of the proton). If each particle has kinetic energy K and they collide head on, why is $2K$ the maximum kinetic energy that can be released (as gamma rays, particle–antiparticle pairs, ...), assuming that the particles survive the collision?

b) As a model of a collider, imagine a collision with one particle at rest (so its total energy is 1) and the other with kinetic energy $2K$ (so its total energy is $2K + 1$), what is the maximum kinetic energy that can be released, again assuming that the particles survive the collision? Hence explain why a collider is more energy-efficient than a normal accelerator. [In SLAC, electrons (with rest mass 0.5 MeV) are accelerated to 25 GeV, so $K = 5 \times 10^4$ in our system of units.]

[Hint: Transform to the zero-momentum frame and use the invariant for momentum and energy often: $E^2 - p^2$. For a particle, $E^2 - p^2 = m^2$, where m is the rest mass, a quantity independent of frame. For a system, $E_{\text{tot}}^2 - p_{\text{tot}}^2$ is not the square of a mass, but it is invariant.]

3 *Resistive grid*

Work out the resistance measured across one resistor in an infinite grid of 1Ω resistors (from the summer vac problem sheet).

4 *Equivalence principle*

General relativity is the **equivalence principle (EP)**: In a small free-floating (or -falling) reference frame, gravity vanishes and you can therefore analyze the world with special relativity. So we can study gravity with special relativity in many tiny frames, combining the results cleverly. For practice with the EP:

a) You drop a sealed bag filled with water. No water comes out while it falls. You poke a hole in the side of the bag and notice that water spurts out. What curve describes its path? Before much water escapes, you drop this punctured bag. Does water come out? If so, what path?

b) Skiing down a frictionless slope, you hold a pendulum. Draw the position of the pendulum and explain your reasoning. How should you stand when you ski?

c) A cart filled with thick corn syrup slides down a frictionless inclined plane. Draw the surface of the corn syrup and explain your reasoning. (I hope we can try this demonstration sometime.)

5 Gravity and clocks

One clock sits on the top floor of the Harvard physics building, at height h ; another sits on the ground floor ($h = 0$). How does gravity affect the relative rates of the clocks? The equivalence principle (EP) can tell you, as I hope this problem will show.

a) You need a frame in which special relativity is valid, but you cannot use the Earth frame because of gravity. To cancel gravity, jump off the building! (You'll be fine 99.9% of the way down.) Immediately, with zero speed, you pass the top clock, which therefore ticks normally, at rate f . When you pass the ground clock, what is your speed, v ? Equivalently, in your frame, how fast is the ground clock moving? Using special relativity, determine the clock's rate from its speed. (First decide: Should it be faster than, slower than, or as fast as the top clock?)

b) For most towers on Earth, $v \ll c$. With that approximation, what is the fractional difference in rates?

c) This problem describes an experiment by Robert Pound and Glen Rebka at the Harvard physics department (R.V. Pound and G.A. Rebka, Jr., 'Apparent weight of photons', *Phys. Rev. Lett.* 4:337–341 (1960)). In their experiment, $h = 23$ m and the clocks were the gamma rays ejected by a nucleus. Using the result from part (b), what is the numerical fractional change in rate?

d) Another perspective on gravity and clocks: A gamma ray from the ground clock loses energy ascending to the top clock: It gets tired. How much energy does one photon of frequency f lose? Energy is frequency says quantum mechanics. What is the fractional change in the photon's frequency (symbolically)? Compare with part (b).

e) Yet another perspective: Photons can be thought of as rocks of mass m ejected from their source with speed c . So, as they fall in a gravity field, they speed up. A photon leaves the top clock, heading for the ground clock. How much has its speed increased when it reaches the ground clock? Call this increase Δv . If the bottom clock were to move downwards with this velocity Δv , what fractional change in frequency would result from the Doppler shift? Compare with part (b). Pound and Rebka used this idea to measure minute changes in frequency – see their paper!