Problem set 4

This sheet is two sided. Wednesday pairs please hand in to my pigeonhole by Wednesday 10am, Friday pairs by Thursday 1pm. Clearly explain your reasoning. Staple your sheets!

1 *Improved petrol*

Drivers want a petrol that yields more joules per kilogram than current petrol does. Discuss the following proposal.

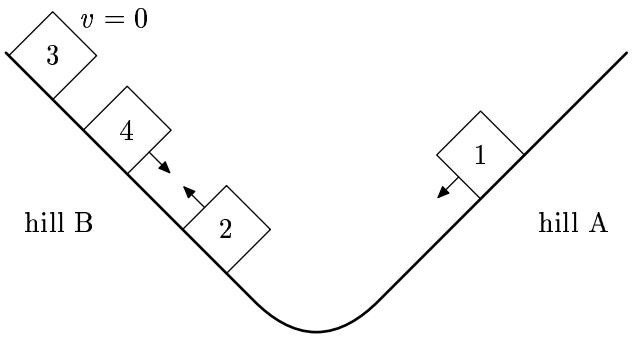
When you accelerate your car from 0 to, say, $15\,\mathrm{m\,s^{-1}}$, the increased kinetic energy, ΔE_1, is supplied by burning a quantity of petrol. Jack is driving in the opposite direction, at $5\,\mathrm{m\,s^{-1}}$. From his point of view, your car was going $5\,\mathrm{m\,s^{-1}}$ and, after accelerating, is going $20\,\mathrm{m\,s^{-1}}$. He measures a change in kinetic energy, ΔE_2; and $\Delta E_2 > \Delta E_1$ (check this assertion). The mass of petrol burned is the same in every reference frame, so Jack measures your petrol to have more energy per unit mass than you measure it to have. So, the proposal is: To increase the energy content of petrol, use a moving reference frame.

2 *Moments of inertia*

Without evaluating any integrals, rank the following objects in order of decreasing moment of inertia: (1) a solid sphere, (2) a thin ring, (3) a spherical shell, and (4) a thin disc. All objects have the same mass and radius and are uniform. For each object, the axis of rotation passes through the centre of mass. For the disc and the ring, the axis is perpendicular to the plane that contains the disc or ring. Explain your rankings.

3 *More skiing*

You ski down hill A and up hill B, then ski backwards down hill B and backwards up hill A (see figure). There is no friction or air resistance, so the cycle repeats forever and ever. Being a skilled skier, you don't need to clutter your hands with poles; instead, from your hand, you dangle a string with a mass at its end. Draw the direction of the string: (1) as you ski down hill A (square with 1 in it), (2) as you ski up hill B (square with 2 in it), (3) when you are momentarily stopped on hill B (square with 3 in it), and (4) as you ski backwards down hill B (square with 4 in it). How does each string direction change if there is slight friction on the slopes?



4 *Falling moon*

The moon is a rock; perhaps large, but it is still a rock. Why doesn't it fall to the earth, as other rocks do? Explain quantitatively, perhaps with one or two diagrams.

5 *Messy collision*

i) A ball comes in from the left and causes a series of collisions; the initial motion is

The number in the circle is the object's mass (in arbitrary units) and the arrow shows the object's velocity (in arbitrary units). All motion is one dimensional, and all collisions are elastic.

Which choice describes the motion after the all the collisions?

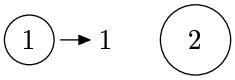
a) 

b) 

c) 

d) 

ii) By transforming to the zero-momentum frame, work out the result of this collision (also one-dimensional and elastic):



Comment on similarities or differences with part i.

6 *Mathematical conservation*

You write a 0 on each vertex of a cube, except for a 1 on one of the vertices. Now you play a game. At each move, you may add 1 to each of two adjacent numbers (adjacent means connected by an edge). Your goal is, using a suitable series of moves, to make all vertex labels be multiples of 3. Is this goal possible? If it is, give the sequence of moves. If it is not, prove the impossibility.

7 *Pendulum*

As a pendulum slowly loses energy, the amplitude of its swing decreases. How does the period change as the amplitude decreases? Is it constant, decreasing, or increasing? Justify your answer.

8 *Centre of mass*

A uniform sphere, of radius r, has a sphere of radius $r/2$ cut out of it. The figure shows a cross section through the sphere. Where is its centre of mass?