

### Problem set 5

Wednesday pairs please hand in to my pigeonhole by Wednesday 10am, Friday pairs by Thursday 1pm. Clearly explain your reasoning.

#### 1 *Estimation: Oblateness of the earth*

Compared to a sphere, the earth is squashed.

- Why? Should the polar radius or the equatorial radius be the larger?
- Which physical quantities determine  $d$ , the difference in radii? How can you combine these quantities into a length (in other words, into an estimate for  $d$ )?
- Use your formula to make a rough numerical estimate of  $d$ , and compare it with actual data.

#### 2 *Moments of inertia*

- What are the dimensions of moment of inertia?
- An object has mass  $M$  and characteristic length  $l$ . The characteristic length is a typical length in the object, such as a radius or diameter. What is its moment of inertia, up to a dimensionless constant? Consider a geometrically similar object that is twice as big as this object, in all its dimensions, and made out of the same material. What is the ratio of moments of inertia:  $I_{\text{bigger}}/I_{\text{smaller}}$ ?
- The moment of inertia of a uniform thin disc is  $MR^2/2$ , about an axis perpendicular to the plane of the disc and through its centre. Perhaps using your results from last week, guess a moment of inertia for a uniform spherical shell with mass  $M$  and radius  $R$  (axis of rotation through the centre). Now calculate it and compare with your guess.

#### 3 *Rolling*

Four objects, made of identical steel, roll down an inclined plane. The objects are (1) a large spherical shell, (2) a large disc, (3) a small solid sphere, and (4) a small ring. The large objects have triple the radius of the small objects. Rank the objects in order of decreasing acceleration down the plane.

#### 4 *Buoyancy*

A solid iron sphere is floating in a bath of mercury. You pour water over the sphere and cover it with water. Does the sphere rise, sink, or stay at the same height?

#### 5 *Quadratics by approximation*

- Use the quadratic formula and your calculator to find the solutions of  $1 + 200000x + x^2 = 0$ . What goes wrong? Why?
- Instead, let's approximate. If  $x$  is near zero, which term can you neglect? Solve the simplified equation to get a first approximation to the smaller root. Call this first approximation  $x_1$ .
- How can you improve your approximation?
- If you know one root, how can you easily find the other root?