Problem set 7

Pairs on Thursday (for this week) please hand in to my pigeonhole by Thursday 11am, Friday pairs by Thursday 7pm. Clearly explain your reasoning.

1 *Atmosphere thickness*

Here is a crude method to estimate the height, H, of the earth's atmosphere. The atmosphere does not end abruptly at H; rather, the density falls gradually to zero. You can think of H as the height at which the density has fallen by a significant fraction. To determine H, mentally launch an air molecule vertically upwards; how high does it reach (if there is no atmosphere in its way)? The height of course depends on the launch velocity. How can you choose a reasonable launch velocity? Get a numerical estimate for the height.

2 *Atmosphere, take 2*

You can also use a more honest method to work out the density versus height in the atmosphere. Assume that the atmosphere has a uniform temperature. Now work out how the density varies with height. (Hint: Consider also how the pressure must vary, and use the ideal gas law to relate pressure and density.) Your density should have the form of the Boltzmann distribution. Coincidence? Discuss.

3 *Return probability in random walks*

From last week: In a one-dimensional random walk, the particle's rms distance from the origin after n steps is $\sqrt{n}$. You can use this result to determine the probability that the particle returns to the origin (the other possibility is that the particle escapes to infinity and never returns). The particle's position is distributed with approximately a Gaussian distribution; the standard deviation is the rms distance $\sqrt{n}$.

Approximate the distribution instead as a rectangle of width $\sqrt{n}$. In other words, replace the Gaussian distribution by a uniform distribution. So p_n, the probability that the particle is at the origin after n steps, is $1/\sqrt{n}$ (give or take a constant). What is the expected number of visits to the origin over all time? What therefore is the probability that the particle returns to the origin?

Extend the argument to two- and three-dimensional random walks. What if anything changes as you go from one to two to three dimensions?

4 *Tricky die (from vac problems)*

You roll a 1000-sided die once per second.

a) How long, on average, between rolls of a `1`? *Answer:* 1000 s

b) Your friend Jane walks up and sees you rolling the die. How long does she have to wait, on average, before a `1` turns up? *Answer:* 1000 s (careful of the gambler's fallacy)

c) How long, on average, between the time that she walked up to you and the time that you *last* rolled a `1`? *Answer:* 1000 s

Combining the answers to parts b and c, we conclude that a `1` turns up every 2000 s, in contradiction to part a. How can you resolve the paradox?

In the kinetic theory, you find the same paradox. A molecule travels on average a distance l (the mean free path) before colliding with another molecule. Observe one of the molecules and be puzzled. How far away, on average, is its next collision? *Answer:* l, because molecules have no memory. How far away, on average, was its last collision? *Answer:* l, because molecules have no memory. So the mean free path should be $2l$.