EXAMPLES 4

A1 *Isothermal atmosphere*

Last week you showed that in an adiabatic atmosphere:

$$T = T_0\left(1 - \beta\frac{z}{H}\right),$$

and

$$T \propto p^\beta,$$

where T_0 is the sea-level temperature and β and H are constants defined by

$$\beta \equiv 1 - \frac{1}{\gamma},$$

and

$$H \equiv \frac{kT_0}{mg}.$$

Show that as $\gamma \to 1$ these results become the isothermal atmosphere that you studied last year. (Note: In the original solutions for sheet 3, I mistakenly left out β in the relation between T and z.)

A2 *Ice skating*

In ice skating the blade must slide on wet ice or else the friction would be too high. One idea for how the ice gets wet is 'pressure melting' due to the high pressure of the ice skate lowering the freezing point. Let's test this idea.

How much does the freezing point drop? The theory is complicated but as a rough guess for the result:

$$\begin{pmatrix}\text{fractional change}\\ \text{in pressure}\end{pmatrix} \sim \begin{pmatrix}\text{fractional change}\\ \text{in } T_{\text{freeze}}\end{pmatrix}$$

or

$$\frac{\Delta p}{p} \sim \frac{\Delta T_{\text{freeze}}}{T_{\text{freeze}}},$$

where the mising constant is hopefully of order 1. Fractional changes are your friend! For water, what p should you use? It has dimensions of energy per volume so construct a reasonable quantity of those dimensions, and estimate the drop in T_{freeze} while ice skating. Is it large enough to melt ice? Whether or not it is, what other effects could melt the ice? Which effects do you think most important?

A3 *Fog*

Fog is tiny water droplets. Why does it form?

How large are the droplets? *Hint:* The mean free path of a light ray is roughly how far you can see on a foggy day.

B1 *Vapor pressure*

Here is vapor pressure data for water:

T (°C)	p *(Pa)*
-10	286.6
0	610.6
10	1227.9
20	2338.5
25	3167.7
30	4239.7
37	6275.5
40	7372.7
60	19918.4
80	47342.8
95	84526.4
100	101325.0
101	105058.0
200	1554405.5

You'll find a more complete version of the data at the course website, so you don't need to type in all the numbers. The purpose of this problem is to explore the relation between vapor pressure and temperature. Perhaps using a spreadsheet and the online data:

(a) First graph p vs T (in Celsius).

(b) The units of temperature and pressure are not natural, so the data is hard to make sense of in this form. Use Kelvin temperature and measure pressures against atmospheric pressure ($p_{\text{atm}} = 101325.0\,\text{Pa}$) by plotting p/p_{atm} vs T (in Kelvin). Notice that $p = p_{\text{atm}}$ at $T = 100\,°\text{C}$. Coincidence?

(c) Notice how widely p/p_{atm} varies, which suggests using a logarithmic scale. Plot $\ln(p/p_{\text{atm}})$ vs T (in Kelvin). Notice that $\ln p$ itself would be a nonsense because you cannot take the logarithm of a Pascal (or any dimension). You need to use a reference pressure, which you have done by using p/p_{atm}.

(d) The graph is still not too straight. As $T \to \infty$, the pressure may asymptote to a constant, although it's hard to tell. One possible solution is to use $1/T$ on the x axis. Plot $\ln(p/p_{\text{atm}})$ vs $1/T$ (in K^{-1}).

(e) Therefore show that

$$p = p_0 e^{-T_0/T},$$

where T_0 is a temperature and p_0 is a pressure (it's not atmospheric pressure). Find the temperature and convert it to an energy per mole, an energy per mass, and an energy (in eV) per molecule. These values should look familiar!

(f) Explain this result using the Boltzmann factor. As part of your explanation, discuss the behavior as $T \to \infty$.

(g) Using the results from sheet 3 (repeated in question A2 this week), what is the temperature drop going up the Rocky Mountains ($h = 3\,\mathrm{km}$)? By what factor does going up the mountain change the vapor pressure of water? What are the consequences of this change for the climate?

D1 *Carnot cycle*

Explain, with suitable diagrams, the four steps of a Carnot cycle. Evaluate the heat flows in each step, and therefore the mechanical work done, and show that a full cycle leaves the entropy unchanged.