Sustainable Energy — without the hot air

Version 3.5.2. December 16, 2008. This Cover-sheet must not appear in the printed book. high-resolution edition.

Equations from *Sustainable Energy - without the hot air*, by David J.C. MacKay, provided for general use. http://www.withouthotair.com/

## Preface

#### 1 Motivations

#### 2 The balance sheet

volume =  $flow \times time$ .

$$flow = \frac{volume}{time}.$$

energy = power  $\times$  time.

kinetic energy =  $\frac{1}{2}mv^2$ .

## 3 Cars

 $\frac{\text{energy used}}{\text{per day}} \ = \ \frac{\text{distance travelled per day}}{\text{distance per unit of fuel}} \times \text{energy per unit of fuel}.$ 

33 miles per imperial gallon  $\simeq 12\, km$  per litre.

 $8\,kWh$  per  $kg\times0.8\,kg$  per litre  $\simeq7\,kWh$  per litre.

energy per day =  $\frac{\text{distance travelled per day}}{\text{distance per unit of fuel}} \times \text{energy per unit of fuel}$  =  $\frac{50 \, \text{km/day}}{12 \, \text{km/litre}} \times 10 \, \text{kWh/litre}$   $\simeq$   $\frac{40 \, \text{kWh/day}}{12 \, \text{km/litre}} \times 10 \, \text{km/litre}$ 

#### 4 Wind

 $power\ per\ person = wind\ power\ per\ unit\ area \times area\ per\ person.$ 

 $2\,\mbox{W/m}^2 \times 4000\,\mbox{m}^2/\mbox{person}\,=\,8000\,\mbox{W}$  per person,

#### 5 Planes

 $\frac{2\times240\,000\,litre}{416\,passengers}\times10\,kWh/litre\simeq12\,000\,kWh~per~passenger.$ 

 $\frac{12\,000\,\text{kWh}}{365\,\text{days}} \simeq 33\,\text{kWh/day}.$ 

## 6 Solar

13 kWh per day per person.

 $20\% \times 110 \, \text{W/m}^2 = 22 \, \text{W/m}^2.$ 

5 kWh per day per person.

# $10\% \times 100 \,\text{W/m}^2 \times 200 \,\text{m}^2 \,\text{per person}$ 50 kWh/day/person.

 $10\% \times 100 \,\text{W/m}^2 = 10 \,\text{W/m}^2.$ 

 $0.5\, \text{W/m}^2 \, imes \, 3000\, \text{m}^2 \, \text{per person} = 36\, \text{kWh/d per person}.$ 

# 7 Heating and cooling

 $4200\,J/litre/^{\circ}C \times 110\,litre \times 40\,^{\circ}C \,\simeq\, 18\,MJ \,\simeq\, 5\,kWh.$ 

# 8 Hydroelectricity

# 9 Light

# 10 Offshore wind

# 11 Gadgets

#### 12 Wave

# 13 Food and farming

$$170 \,\mathrm{kg} \times \frac{3 \,\mathrm{kWh/d}}{65 \,\mathrm{kg}} \,\simeq \, 8 \,\mathrm{kWh/d}.$$

# 15 Stuff

#### 16 Geothermal

#### 17 Public services

#### 18 Can we live on renewables?

## 19 Every BIG helps

# 20 Better transport

## 21 Smarter heating

 $power~used = \frac{average~temperature~difference \times leakiness~of~building}{efficiency~of~heating~system}.$ 

average temperature difference  $\times$  leakiness of building

 $9 \,^{\circ}\text{C} \times 7.7 \,\text{kWh/d/}^{\circ}\text{C} \simeq 70 \,\text{kWh/d}.$ 

power used = 
$$\frac{9 \,^{\circ}\text{C} \times 7.7 \,\text{kWh/d/^{\circ}\text{C}}}{0.9} = 77 \,\text{kWh/d}.$$

 $power~used = \frac{average~temperature~difference \times leakiness~of~building}{efficiency~of~heating~system}.$ 

# 22 Efficient electricity use

## 23 Sustainable fossil fuels?

#### 24 Nuclear?

 $\frac{4.5 \, \text{billion tons per planet}}{162 \, \text{tons uranium per GW-year}} = 28 \, \text{million GW-years per planet}.$ 

2.8 million GW-years / 1600 years = 1750 GW,

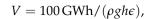
carbon intensity 
$$= \frac{300 \times 10^9 \, g}{10^6 \, kW(e) \times 220\,000 \, h}$$
 
$$= 1.4 \, g/kWh(e),$$

25 Living on other countries' renewables?

## 26 Fluctuations and storage

$$84\,\text{MW/h} imes rac{33\,000\,\text{MW}}{745\,\text{MW}} = 3700\,\text{MW/h},$$

 $10 \,\text{GW} \times (5 \times 24 \,\text{h}) = 1200 \,\text{GWh}.$ 



# 27 Five energy plans for Britain

## 28 Putting costs in perspective

#### 29 What to do now

30 Energy plans for Europe, America, and the World

$$\frac{1}{5} \times 10\% \times 9000 \, m^2 \times 2 \, W/m^2 = 360 \, \, W/m^2$$

 $5 \, \text{W/m}^2 \times 450 \, \text{m}^2 = 54 \, \text{kWh/d}$  per person.

## 31 The last thing we should talk about

## 32 Saying yes

## A Cars II

kinetic energy =  $\frac{1}{2}mv^2$ .

 $\frac{1}{2}mv^2 \simeq 390\,000\,\mathrm{J} \simeq 0.1\,\mathrm{kWh}.$ 

$$\frac{\text{kinetic energy}}{\text{time between braking events}} = \frac{\frac{1}{2}m_{c}v^{2}}{d/v} = \frac{\frac{1}{2}m_{c}v^{3}}{d},$$
(A.1)

 $c_{\rm d}A_{\rm car}$ 

 $mass = density \times volume$ 

$$\frac{1}{2}m_{\rm air}v^2 = \frac{1}{2}\rho Avt\,v^2,$$

$$\frac{\frac{1}{2}\rho Avtv^2}{t} = \frac{1}{2}\rho Av^3.$$

power going into brakes + power going into swirling air 
$$= \frac{1}{2}m_{c}v^{3}/d + \frac{1}{2}\rho Av^{3}. \tag{A.2}$$

 $A_{\rm car} = 2 \,\mathrm{m}\,\mathrm{wide} \times 1.5 \,\mathrm{m}\,\mathrm{high} = 3 \,\mathrm{m}^2$ 

$$d^* = \frac{m_c}{\rho c_d A_{car}} = \frac{1000 \text{ kg}}{1.3 \text{ kg/m}^3 \times \frac{1}{3} \times 3 \text{ m}^2} = 750 \text{ m}.$$

total power of car  $\simeq 4 \left[ \frac{1}{2} m_{\rm c} v^3 / d + \frac{1}{2} \rho A v^3 \right]$ .

$$A = c_{\rm d} A_{\rm car} = 1$$

$$4 \times \frac{1}{2} \rho A v^3 = 2 \times 1.3 \, \text{kg/m}^3 \times 1 \, \text{m}^2 \times (31 \, \text{m/s})^3 = 80 \, \text{kW}.$$

$$4 \times \frac{1}{2} \rho A v^3,$$

energy per distance  $= 4 \times \frac{1}{2} \rho A v^2$ .

 $A = c_{\rm d} A_{\rm car}$ 

 $4 \times \tfrac{1}{2} \rho A v^2$ 

 $\frac{\text{energy per distance of bike}}{\text{energy per distance of car}} = \frac{c_{\rm d}^{\rm bike} A_{\rm bike} v_{\rm bike}^2}{c_{\rm d}^{\rm car} A_{\rm car} v_{\rm car}^2}.$ 

$$\frac{A_{\text{bike}}}{A_{\text{car}}} = \frac{1}{4}.$$

$$\frac{c_{\rm d}^{\rm bike}}{c_{\rm d}^{\rm car}} = \frac{1}{1/3}$$

$$\frac{v_{\text{bike}}}{v_{\text{car}}} = \frac{1}{5}.$$

$$\begin{array}{ll} \frac{\text{energy-per-distance of bike}}{\text{energy-per-distance of car}} &=& \left(\frac{c_{\rm d}^{\rm bike}}{c_{\rm d}^{\rm car}} \frac{A_{\rm bike}}{A_{\rm car}}\right) \left(\frac{v_{\rm bike}}{v_{\rm car}}\right)^2 \\ &=& \left(\frac{3}{4}\right) \times \left(\frac{1}{5}\right)^2 \\ &=& \frac{3}{100} \end{array}$$

force  $\times$  velocity =  $(100 \text{ newtons}) \times (31 \text{ m/s}) = 3100 \text{ W/m}^2$ ;

$$C_{\rm rr}m_{\rm c}g=\frac{1}{2}\rho c_{\rm d}Av^2,$$

$$v = \sqrt{2 \frac{C_{\rm rr} m_{\rm c} g}{\rho c_{\rm d} A}} = 7 \,\mathrm{m/s} = 16 \,\mathrm{miles}$$
 per hour.

 $v = 33 \,\mathrm{m/s} = 74 \,\mathrm{miles}$  per hour.

 $v = 12 \,\mathrm{m/s} = 26 \,\mathrm{miles}$  per hour.

 $c_{\rm d}A=0.3\,{\rm m}^2$ 

 $mass = density \times volume$ 

$$\frac{1}{2}mv^2 = \frac{1}{2}\rho Avt \, v^2 = \frac{1}{2}\rho Atv^3. \tag{B.1}$$

$$\frac{\frac{1}{2}mv^2}{t} = \frac{1}{2}\rho A v^3. {(B.2)}$$

$$\frac{1}{2}\rho v^3 = \frac{1}{2}1.3 \,\mathrm{kg/m^3} \times (6 \,\mathrm{m/s})^3 = 140 \,\mathrm{W/m^2}. \tag{B.3}$$

efficiency factor  $\times$  power per unit area  $\times$  area

$$= 50\% \times \frac{1}{2}\rho v^3 \times \frac{\pi}{4}d^2 \tag{B.4}$$

$$= 50\% \times 140 \,\text{W/m}^2 \times \frac{\pi}{4} (25 \,\text{m})^2 \tag{B.5}$$

$$= 34 \,\mathrm{kW}.$$
 (B.6)

$$\frac{\text{power per windmill (B.4)}}{\text{land area per windmill}} = \frac{\frac{1}{2}\rho v^3 \frac{\pi}{8} d^2}{(5d)^2}$$
(B.7)

$$= \frac{\pi}{200} \frac{1}{2} \rho v^3 \tag{B.8}$$

$$= 0.016 \times 140 \,\text{W/m}^2$$

$$= 2.2 \,\text{W/m}^2.$$
(B.9)
(B.10)

$$= 2.2 \,\mathrm{W/m^2}.$$
 (B.10)

$$v(z) = v_{10} \left(\frac{z}{10\,\mathrm{m}}\right)^{\alpha},$$

$$v(z) = v_{\text{ref}} \frac{\log(z/z_0)}{\log(z_{\text{ref}}/z_0)},$$

## C Planes II

force = rate of change of momentum, (C.1)

force exerted on A by B = - force exerted on B by A.

(C.2)

$$m_{\text{sausage}} = \text{density} \times \text{volume} = \rho v t A_{\text{s}}.$$
 (C.3)

$$mass \times velocity = m_{sausage}u = \rho vt A_s u. \tag{C.4}$$

mgt. (C.5)

$$\rho vt A_{s} u = mgt, \tag{C.6}$$

$$u = \frac{mg}{\rho v A_{\rm s}}.$$

$$P_{\text{lift}} = \frac{\text{kinetic energy of sausage}}{\text{time}}$$
 (C.7)  
=  $\frac{1}{t} \frac{1}{2} m_{\text{sausage}} u^2$  (C.8)

$$= \frac{1}{t} \frac{1}{2} m_{\text{sausage}} u^2 \tag{C.8}$$

$$= \frac{1}{2t}\rho v t A_{\rm s} \left(\frac{mg}{\rho v A_{\rm s}}\right)^{2}$$

$$= \frac{1}{2} \frac{(mg)^{2}}{\rho v A_{\rm s}}.$$
(C.9)

$$= \frac{1}{2} \frac{(mg)^2}{\rho v A_s}.$$
 (C.10)

$$P_{\text{total}} = P_{\text{drag}} + P_{\text{lift}} \tag{C.11}$$

$$P_{\text{total}} = P_{\text{drag}} + P_{\text{lift}}$$
 (C.11)  
=  $\frac{1}{2}c_{\text{d}}\rho A_{\text{p}}v^{3} + \frac{1}{2}\frac{(mg)^{2}}{\rho v A_{\text{s}}}$ , (C.12)

$$\frac{\text{energy}}{\text{distance}}\Big|_{\text{ideal}} = \frac{P_{\text{total}}}{v} = \frac{1}{2}c_{\text{d}}\rho A_{\text{p}}v^2 + \frac{1}{2}\frac{(mg)^2}{\rho v^2 A_{\text{s}}},\tag{C.13}$$

 $\tfrac{1}{2}c_{\rm d}\rho A_{\rm p}v^2$ 

 $\tfrac{1}{2} \frac{(mg)^2}{\rho v^2 A_s}$ 

 $\tfrac{1}{2}c_{\rm d}\rho A_{\rm p}v^2$ 

 $\tfrac{1}{2} \frac{(mg)^2}{\rho v^2 A_s}$ 

$$\frac{\text{energy}}{\text{distance}} = \frac{1}{\epsilon} \left( \frac{1}{2} c_{\text{d}} \rho A_{\text{p}} v^2 + \frac{1}{2} \frac{(mg)^2}{\rho v^2 A_{\text{s}}} \right). \tag{C.14}$$

 $\tfrac{1}{2}c_{\rm d}\rho A_{\rm p}v^2$ 

 $\tfrac{1}{2} \tfrac{(mg)^2}{\rho v^2 A_s}$ 

$$c_{\rm d}\rho A_{\rm p}v^2 = \frac{(mg)^2}{\rho v^2 A_{\rm s}},$$
 (C.15)

$$\rho v_{\rm opt}^2 = \frac{mg}{\sqrt{c_{\rm d}A_{\rm p}A_{\rm s}}},\tag{C.16}$$

force = 
$$\frac{\text{energy}}{\text{distance}}\Big|_{\text{ideal}} = \frac{1}{2}c_{\text{d}}\rho A_{\text{p}}v^2 + \frac{1}{2}\frac{(mg)^2}{\rho v^2 A_{\text{s}}}$$
 (C.17)

$$= c_{\rm d}\rho A_{\rm p} v_{\rm opt}^2 \tag{C.18}$$

$$= c_{\rm d}\rho A_{\rm p} v_{\rm opt}^2$$

$$= c_{\rm d}\rho A_{\rm p} \frac{mg}{\rho(c_{\rm d}A_{\rm p}A_{\rm s})^{1/2}}$$
(C.18)
$$= c_{\rm d}\rho A_{\rm p} \frac{mg}{\rho(c_{\rm d}A_{\rm p}A_{\rm s})^{1/2}}$$

$$= \left(\frac{c_{\rm d}A_{\rm p}}{A_{\rm s}}\right)^{1/2} mg. \tag{C.20}$$

$$f_A = \frac{A_p}{A_s}. (C.21)$$

force = 
$$(c_{\rm d}f_A)^{1/2}(mg)$$
. (C.22)

$$(c_{\rm d}f_A)^{1/2} mg = 0.036 mg = 130 \,\text{kN}.$$
 (C.23)

transport cost 
$$= \frac{1}{\epsilon} \frac{\text{force}}{\text{mass}}$$
 (C.24)  
$$= \frac{1}{\epsilon} \frac{(c_{\text{d}}f_A)^{1/2} mg}{m}$$
 (C.25)  
$$= \frac{(c_{\text{d}}f_A)^{1/2}}{\epsilon} g.$$
 (C.26)

$$= \frac{1}{\epsilon} \frac{(c_{\rm d}f_A)^{1/2}mg}{m} \tag{C.25}$$

$$= \frac{(c_{\rm d}f_A)^{1/2}}{\epsilon}g. \tag{C.26}$$

0.4 kWh/ton-km.

transport efficiency (passenger-km per litre of fuel)

= number of passengers 
$$\times \frac{\text{energy per litre}}{\frac{\text{thrust}}{\epsilon}}$$
 (C.27)  
= number of passengers  $\times \frac{\epsilon \times \text{energy per litre}}{\text{thrust}}$  (C.28)

= number of passengers 
$$\times \frac{\epsilon \times \text{energy per litre}}{\text{thrust}}$$
 (C.28)

$$= 400 \times \frac{1}{3} \frac{38 \text{ MJ/litre}}{200 000 \text{ N}}$$

$$= 25 \text{ passenger-km per litre}$$
(C.29)

$$range = v_{opt} \frac{energy}{power} = \frac{energy \times \epsilon}{force}.$$
 (C.31)

range = 
$$\frac{\text{energy }\epsilon}{\text{force}} = \frac{Cm\epsilon f_{\text{fuel}}}{(c_{\text{d}}f_A)^{1/2}(mg)} = \frac{\epsilon f_{\text{fuel}}}{(c_{\text{d}}f_A)^{1/2}} \frac{C}{g}.$$
 (C.32)

 $\left(\frac{\epsilon f_{\text{fuel}}}{(c_{\text{d}}f_A)^{1/2}}\right)$ 

 $\frac{C}{g}$ ,

$$d_{\text{Fuel}} = \frac{C}{g} = 4000 \,\text{km}.$$
 (C.33)

 $\left(\frac{\epsilon f_{\text{fuel}}}{(c_{\text{d}}f_A)^{1/2}}\right)$ 

 $(c_{\rm d}f_A)^{1/2}\simeq 1/20$ 

 $v^2 \sim mg/(\rho(c_{\rm d}A_{\rm p}A_{\rm s})^{1/2})$ 

0.4 kWh/ton-km.

$$\frac{(c_{\rm d}f_A)^{1/2}}{\epsilon}g,\tag{C.34}$$

 $m_{\mathrm{total}} = \rho V$ 

$$F = \frac{1}{2}c_{\rm d}A\rho v^2,\tag{C.35}$$

$$\frac{F}{\epsilon m_{\text{total}}} = \frac{\frac{1}{2} c_{\text{d}} A \rho v^2}{\epsilon \rho_3^2 A L}$$

$$= \frac{3}{4 \epsilon} c_{\text{d}} \frac{v^2}{L}$$
(C.36)

$$= \frac{3}{4\epsilon}c_{\rm d}\frac{v^2}{L} \tag{C.37}$$

$$\frac{F}{\epsilon m_{\text{total}}} = 3 \times 0.03 \frac{(22 \,\text{m/s})^2}{400 \,\text{m}} = 0.1 \,\text{m/s}^2 = 0.03 \,\text{kWh/t-km}.$$

## D Solar II

## E Heating II

power loss = area  $\times$  *U*  $\times$  temperature difference.

$$u_{\text{series combination}} = 1 / \left(\frac{1}{u_1} + \frac{1}{u_2}\right).$$

power (watts) = 
$$C \frac{N}{1 \text{ h}} V(\text{m}^3) \Delta T(\text{K})$$
 (E.1)

$$= (1.2 \,\mathrm{kJ/m^3/K}) \frac{N}{3600 \,\mathrm{s}} V(\mathrm{m^3}) \Delta T(\mathrm{K})$$
 (E.2)

$$= \frac{1}{3}NV\Delta T. \tag{E.3}$$

energy loss = area × U × ( $\Delta T$  × duration),

 $\frac{1}{3}NV \times (\Delta T \times \text{duration}).$ 

Something  $\times$  ( $\Delta T \times$  duration),

energy lost = leakiness  $\times$  temperature demand.

energy consumed = energy delivered/coefficient of performance.

322 W/m²/°C × 120 degree-hours  $\simeq$  39 kWh.

 $7.7\,kWh/d/^{\circ}C \times 2866\,degree-days/y/(365\,days/y) = 61\,kWh/d.$ 

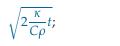
$$1 / \left( \frac{1}{2.2} + \frac{1}{1.7} \right) \simeq 1 \, \text{W/m}^2 / \text{K}.$$

efficiency = 
$$\frac{T_2}{T_2 - T_1}$$
.

efficiency = 
$$\frac{T_2}{T_1 - T_2}$$
.

$$\begin{array}{ll} \text{mass} & = & \frac{\text{energy}}{\text{heat capacity} \times \text{temperature difference}} \\ & = & \frac{24 \times 30 \times 3.6 \, \text{MJ}}{(820 \, \text{J/kg/°C})(50 \, ^{\circ}\text{C} - 16 \, ^{\circ}\text{C})} \\ & = & 100 \, 000 \, \text{kg,} \end{array}$$

$$\frac{1}{\sqrt{4\pi\kappa t}}\exp\left(-\frac{x^2}{4(\kappa/(C\rho))t}\right)$$



 $5\,\mbox{W/m}^2 \times 160\,\mbox{m}^2 = 800\,\mbox{W/m}^2 = 19\,\mbox{kWh/d}$  per person.

Flux = 
$$\kappa \times \frac{\Delta T}{h} = 3 \text{ W/m}^2$$
.

$$\frac{\partial T(z,t)}{\partial t} = \frac{\kappa}{C_{\rm V}} \frac{\partial^2 T(z,t)}{\partial z^2}.$$
 (E.4)

$$T(0,t) = T_{\text{surface}}(t) = T_{\text{average}} + A\cos(\omega t),$$
 (E.5)

$$T(z,t) = T_{\text{average}} + A e^{-z/z_0} \cos(\omega t - z/z_0), \tag{E.6}$$

$$z_0 = \sqrt{\frac{2\kappa}{C_V \omega}}. (E.7)$$

$$\kappa \frac{\partial T}{\partial z} = \kappa \frac{A}{z_0} \sqrt{2} e^{-z/z_0} \sin(\omega t - z/z_0 - \pi/4). \tag{E.8}$$

$$\kappa \frac{A}{z_0} \sqrt{2} = A \sqrt{C_V \kappa \omega}. \tag{E.9}$$

## F Waves II

$$v = \frac{gT}{2\pi},$$

$$P_{\rm potential} \simeq m^* g \bar{h} / T,$$
 (F.1)

$$P_{\rm potential} \simeq \frac{1}{2} \rho h \frac{\lambda}{2} g h / T.$$
 (F.2)

$$P_{\text{potential}} \simeq \frac{1}{4} \rho g h^2 v.$$
 (F.3)

$$P_{\text{total}} \simeq \frac{1}{2} \rho g h^2 v.$$
 (F.4)

$$P_{\text{total}} = \frac{1}{4} \rho g h^2 v. \tag{F.5}$$

$$P_{\text{total}} = \frac{1}{4} \rho g h^2 v = 40 \,\text{kW/m}.$$
 (F.6)

 $\frac{2\rho hgh}{6 \text{ hours}}$ 

power per unit area of tide-pool  $\simeq 3 \, \text{W/m}^2$ .

$$\rho g^{3/2} \sqrt{dh^2/2}$$
. (G.1)

$$v = \sqrt{gd}. (G.2)$$

$$U = vh/d. (G.3)$$

$$K_{\rm BV} = \frac{1}{2} \rho A U^3,\tag{G.4}$$

$$\frac{1}{4}\rho gh^2\lambda. \tag{G.5}$$

$$power = \frac{1}{2}(\rho g h^2 \lambda) \times w/T = \frac{1}{2}\rho g h^2 v \times w,$$
(G.6)

power = 
$$\rho g h^2 \sqrt{g d} \times w/2 = \rho g^{3/2} \sqrt{d} h^2 \times w/2$$
. (G.7)

$$K_{\rm BV} = \frac{1}{2}\rho A U^3 = \frac{1}{2}\rho w d(vh/d)^3 = \rho \left(g^{3/2}/\sqrt{d}\right) h^3 \times w/2. \tag{G.8}$$

$$\frac{K_{\text{BV}}}{\text{power}} = \frac{\rho w \left(g^{3/2} / \sqrt{d}\right) h^3}{\rho g^{3/2} h^2 \sqrt{d} w} = \frac{h}{d}.$$
(G.9)

 $\frac{\text{power per tidemill}}{\text{area per tidemill}} \ \ = \ \ \frac{\pi}{200} \frac{1}{2} \rho U^3.$ 

 $\tfrac{\pi}{200} \tfrac{1}{2} \rho U^3$ 

$$b/\epsilon_{\rm p} = \epsilon_{\rm g}(b+2h).$$

 $\epsilon = \epsilon_{\rm g} \epsilon_{\rm p}$ 

$$b=2h\frac{\epsilon}{1-\epsilon}.$$

$$\left(\frac{1}{2}\rho g\epsilon_{\rm g}(b+2h)^2-\frac{1}{2}\rho g\frac{1}{\epsilon_{\rm p}}b^2\right)/T$$
,

$$\left(\frac{1}{1-\epsilon}\right)$$
,

## H Stuff II

## I Quick reference

## J Populations and areas

## K UK energy history