## Sustainable Energy - without the hot air

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Equations from Sustainable Energy - without the hot air, by David J.C. MacKay, provided for general use. http://www.withouthotair.com/

## Preface

## 1 Motivations

2 The balance sheet

$$
\text { volume }=\text { flow } \times \text { time }
$$

$$
\text { flow }=\frac{\text { volume }}{\text { time }} .
$$

$$
\text { energy }=\text { power } \times \text { time }
$$

$$
\text { kinetic energy }=\frac{1}{2} m v^{2} .
$$

3 Cars
$\underset{\text { per day }}{\text { energy used }}=\frac{\text { distance travelled per day }}{\text { distance per unit of fuel }} \times$ energy per unit of fuel.

33 miles per imperial gallon $\simeq 12 \mathrm{~km}$ per litre.

8 kWh per $\mathrm{kg} \times 0.8 \mathrm{~kg}$ per litre $\simeq 7 \mathrm{kWh}$ per litre.

$$
\begin{aligned}
\text { energy per day } & =\frac{\text { distance travelled per day }}{\text { distance per unit of fuel }} \times \text { energy per unit of fuel } \\
& =\frac{50 \mathrm{~km} / \text { day }}{12 \mathrm{~km} / \text { litre }} \times 10 \mathrm{kWh} / \text { litre } \\
& \simeq 40 \mathrm{kWh} / \text { day. }
\end{aligned}
$$

4 Wind
power per person $=$ wind power per unit area $\times$ area per person.

$$
2 \mathrm{~W} / \mathrm{m}^{2} \times 4000 \mathrm{~m}^{2} / \text { person }=8000 \mathrm{~W} \text { per person, }
$$

## 5 Planes

$\frac{2 \times 240000 \text { litre }}{416 \text { passengers }} \times 10 \mathrm{kWh} /$ litre $\simeq 12000 \mathrm{kWh}$ per passenger.

$$
\frac{12000 \mathrm{kWh}}{365 \text { days }} \simeq 33 \mathrm{kWh} / \text { day } .
$$

6 Solar

13 kWh per day per person.

5 kWh per day per person.
$10 \% \times 100 \mathrm{~W} / \mathrm{m}^{2} \times 200 \mathrm{~m}^{2}$ per person
$10 \% \times 100 \mathrm{~W} / \mathrm{m}^{2}=10 \mathrm{~W} / \mathrm{m}^{2}$.
$0.5 \mathrm{~W} / \mathrm{m}^{2} \times 3000 \mathrm{~m}^{2}$ per person $=36 \mathrm{kWh} /$ d per person.

## 7 Heating and cooling

$4200 \mathrm{~J} /$ litre $/{ }^{\circ} \mathrm{C} \times 110$ litre $\times 40^{\circ} \mathrm{C} \simeq 18 \mathrm{MJ} \simeq 5 \mathrm{kWh}$.

8 Hydroelectricity

$$
9 \text { Light }
$$

10 Offshore wind

11 Gadgets

12 Wave

13 Food and farming

$$
170 \mathrm{~kg} \times \frac{3 \mathrm{kWh} / \mathrm{d}}{65 \mathrm{~kg}} \simeq 8 \mathrm{kWh} / \mathrm{d}
$$

14 Tide

## 15 Stuff

16 Geothermal

## 17 Public services

18 Can we live on renewables?

## 19 Every BIG helps

## 20 Better transport

21 Smarter heating

$$
\text { power used }=\frac{\text { average temperature difference } \times \text { leakiness of building }}{\text { efficiency of heating system }} .
$$

average temperature difference $\times$ leakiness of building
$9^{\circ} \mathrm{C} \times 7.7 \mathrm{kWh} / \mathrm{d} /{ }^{\circ} \mathrm{C} \simeq 70 \mathrm{kWh} / \mathrm{d}$.

$$
\text { power used }=\frac{9^{\circ} \mathrm{C} \times 7.7 \mathrm{kWh} / \mathrm{d} /{ }^{\circ} \mathrm{C}}{0.9}=77 \mathrm{kWh} / \mathrm{d}
$$

$$
\text { power used }=\frac{\text { average temperature difference } \times \text { leakiness of building }}{\text { efficiency of heating system }} .
$$

## 22 Efficient electricity use

23 Sustainable fossil fuels?

24 Nuclear?

$$
\frac{4.5 \text { billion tons per planet }}{162 \text { tons uranium per GW-year }}=28 \text { million GW-years per planet. }
$$

2.8 million GW-years / 1600 years $=1750 \mathrm{GW}$,

$$
\begin{aligned}
\begin{array}{c}
\text { carbon intensity } \\
\text { associated with construction }
\end{array} & =\frac{300 \times 10^{9} \mathrm{~g}}{10^{6} \mathrm{~kW}(\mathrm{e}) \times 220000 \mathrm{~h}} \\
& =1.4 \mathrm{~g} / \mathrm{kWh}(\mathrm{e})
\end{aligned}
$$

25 Living on other countries' renewables?

26 Fluctuations and storage
$84 \mathrm{MW} / \mathrm{h} \times \frac{33000 \mathrm{MW}}{745 \mathrm{MW}}=3700 \mathrm{MW} / \mathrm{h}$,
$10 \mathrm{GW} \times(5 \times 24 \mathrm{~h})=1200 \mathrm{GWh}$.

$$
V=100 \mathrm{GWh} /(\rho g h \epsilon),
$$

27 Five energy plans for Britain

28 Putting costs in perspective

29 What to do now

30 Energy plans for Europe, America, and the World

$$
\frac{1}{5} \times 10 \% \times 9000 \mathrm{~m}^{2} \times 2 \mathrm{~W} / \mathrm{m}^{2}=360 \mathrm{~W} / \mathrm{m}^{2}
$$

$5 \mathrm{~W} / \mathrm{m}^{2} \times 450 \mathrm{~m}^{2}=54 \mathrm{kWh} / \mathrm{d}$ per person.

31 The last thing we should talk about

32 Saying yes

A Cars II
kinetic energy $=\frac{1}{2} m v^{2}$.

$$
\frac{1}{2} m v^{2} \simeq 390000 \mathrm{~J} \simeq 0.1 \mathrm{kWh} .
$$

$$
\begin{equation*}
\frac{\text { kinetic energy }}{\text { time between braking events }}=\frac{\frac{1}{2} m_{\mathrm{c}} v^{2}}{d / v}=\frac{\frac{1}{2} m_{\mathrm{c}} v^{3}}{d} \tag{A.1}
\end{equation*}
$$

$c_{\mathrm{d}} A_{\text {car }}$

$$
\text { mass }=\text { density } \times \text { volume }
$$

$m_{\mathrm{air}}=\rho A v t$

$$
\frac{1}{2} m_{\mathrm{air}} v^{2}=\frac{1}{2} \rho A v t v^{2}
$$

$$
\frac{\frac{1}{2} \rho A v t v^{2}}{t}=\frac{1}{2} \rho A v^{3} .
$$

power going into brakes + power going into swirling air
$=\frac{1}{2} m_{\mathcal{C}} v^{3} / d+\frac{1}{2} \rho A v^{3}$.

$$
\begin{equation*}
=\frac{1}{2} m_{\mathrm{c}} v^{3} / d+\frac{1}{2} \rho A v^{3} . \tag{A.2}
\end{equation*}
$$

$\left(m_{\mathrm{c}} / d\right) /(\rho A)$.

$$
A_{\mathrm{car}}=2 \mathrm{~m} \text { wide } \times 1.5 \mathrm{~m} \text { high }=3 \mathrm{~m}^{2}
$$

$$
c_{d}=1 / 3
$$

$$
d^{*}=\frac{m_{\mathrm{c}}}{\rho c_{\mathrm{d}} A_{\mathrm{car}}}=\frac{1000 \mathrm{~kg}}{1.3 \mathrm{~kg} / \mathrm{m}^{3} \times \frac{1}{3} \times 3 \mathrm{~m}^{2}}=750 \mathrm{~m} .
$$

$$
\text { total power of car } \simeq 4\left[\frac{1}{2} m_{\mathrm{c}} v^{3} / d+\frac{1}{2} \rho A v^{3}\right]
$$

$$
A=c_{\mathrm{d}} A_{\mathrm{car}}=1
$$

$$
4 \times \frac{1}{2} \rho A v^{3}=2 \times 1.3 \mathrm{~kg} / \mathrm{m}^{3} \times 1 \mathrm{~m}^{2} \times(31 \mathrm{~m} / \mathrm{s})^{3}=80 \mathrm{~kW} .
$$

$$
4 \times \frac{1}{2} \rho A v^{3},
$$

$$
\text { energy per distance }=4 \times \frac{1}{2} \rho A v^{2}
$$

$$
A=c_{\mathrm{d}} A_{\mathrm{car}}
$$

$$
\frac{\text { energy per distance of bike }}{\text { energy per distance of car }}=\frac{c_{\mathrm{d}}^{\text {bike }} A_{\text {bike }} v_{\text {bike }}^{2}}{c_{\mathrm{d}}^{\text {car }} A_{\text {car }} v_{\mathrm{car}}^{2}} .
$$

$$
\frac{A_{\mathrm{bike}}}{A_{\mathrm{car}}}=\frac{1}{4} .
$$

$$
\frac{c_{d}^{\text {bike }}}{c_{d}^{\text {car }}}=\frac{1}{1 / 3}
$$

$$
\frac{v_{\mathrm{bike}}}{v_{\mathrm{car}}}=\frac{1}{5}
$$

$$
\begin{aligned}
\frac{\text { energy-per-distance of bike }}{\text { energy-per-distance of car }} & =\left(\frac{c_{\mathrm{d}}^{\text {bike }}}{c_{\mathrm{d}}^{\text {car }}} \frac{A_{\text {bike }}}{A_{\text {car }}}\right)\left(\frac{v_{\text {bike }}}{v_{\text {car }}}\right)^{2} \\
& =\left(\frac{3}{4}\right) \times\left(\frac{1}{5}\right)^{2} \\
& =\frac{3}{100}
\end{aligned}
$$

$c_{\mathrm{d}} A_{\text {car }}=1$
$c_{\mathrm{d}} A_{\text {train }}=11$

$$
\text { force } \times \text { velocity }=(100 \text { newtons }) \times(31 \mathrm{~m} / \mathrm{s})=3100 \mathrm{~W} / \mathrm{m}^{2} \text {; }
$$

$$
C_{\mathrm{rr}} m_{\mathrm{c}} g=\frac{1}{2} \rho c_{\mathrm{d}} A v^{2}
$$

$$
v=\sqrt{2 \frac{C_{\mathrm{rr}} m_{\mathrm{c}} g}{\rho c_{\mathrm{d}} A}}=7 \mathrm{~m} / \mathrm{s}=16 \text { miles per hour. }
$$

$$
v=33 \mathrm{~m} / \mathrm{s}=74 \text { miles per hour. }
$$

$$
v=12 \mathrm{~m} / \mathrm{s}=26 \text { miles per hour. }
$$

$$
c_{\mathrm{d}} A=0.3 \mathrm{~m}^{2}
$$

B Wind II

$$
\text { mass }=\text { density } \times \text { volume }
$$

$$
\begin{equation*}
\frac{1}{2} m v^{2}=\frac{1}{2} \rho A v t v^{2}=\frac{1}{2} \rho A t v^{3} . \tag{B.1}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\frac{1}{2} m v^{2}}{t}=\frac{1}{2} \rho A v^{3} . \tag{B.2}
\end{equation*}
$$

$$
\begin{equation*}
\frac{1}{2} \rho v^{3}=\frac{1}{2} 1.3 \mathrm{~kg} / \mathrm{m}^{3} \times(6 \mathrm{~m} / \mathrm{s})^{3}=140 \mathrm{~W} / \mathrm{m}^{2} . \tag{B.3}
\end{equation*}
$$

$$
\begin{align*}
& \text { efficiency factor } \times \text { power per unit area } \times \text { area } \\
= & 50 \% \times \frac{1}{2} \rho v^{3} \times \frac{\pi}{4} d^{2} \\
= & 50 \% \times 140 \mathrm{~W} / \mathrm{m}^{2} \times \frac{\pi}{4}(25 \mathrm{~m})^{2}  \tag{B.5}\\
= & 34 \mathrm{~kW} . \tag{B.6}
\end{align*}
$$

$$
\begin{align*}
\frac{\text { power per windmill (B.4) }}{\text { land area per windmill }} & =\frac{\frac{1}{2} \rho v^{3} \frac{\pi}{8} d^{2}}{(5 d)^{2}}  \tag{B.7}\\
& =\frac{\pi}{200} \frac{1}{2} \rho v^{3}  \tag{B.8}\\
& =0.016 \times 140 \mathrm{~W} / \mathrm{m}^{2}  \tag{B.9}\\
& =2.2 \mathrm{~W} / \mathrm{m}^{2} . \tag{B.10}
\end{align*}
$$

$$
v(z)=v_{10}\left(\frac{z}{10 \mathrm{~m}}\right)^{\alpha},
$$

$$
v(z)=v_{\text {ref }} \frac{\log \left(z / z_{0}\right)}{\log \left(z_{\text {ref }} / z_{0}\right)},
$$

C Planes II

$$
\begin{equation*}
\rho v t A_{\mathrm{s}} u=m g t \tag{C.6}
\end{equation*}
$$

$$
u=\frac{m g}{\rho v A_{\mathrm{s}}}
$$

$$
\begin{align*}
P_{\text {lift }} & =\frac{\text { kinetic energy of sausage }}{\text { time }}  \tag{C.7}\\
& =\frac{1}{t} \frac{1}{2} m_{\text {sausage }} u^{2}  \tag{C.8}\\
& =\frac{1}{2 t} \rho v t A_{\mathrm{s}}\left(\frac{m g}{\rho v A_{\mathrm{s}}}\right)^{2}  \tag{C.9}\\
& =\frac{1}{2} \frac{(m g)^{2}}{\rho v A_{\mathrm{s}}} . \tag{C.10}
\end{align*}
$$

$$
\begin{align*}
P_{\text {total }} & =P_{\mathrm{drag}}+P_{\mathrm{lift}}  \tag{С.11}\\
& =\frac{1}{2} c_{\mathrm{d}} \rho A_{\mathrm{p}} v^{3}+\frac{1}{2} \frac{(m g)^{2}}{\rho v A_{\mathrm{s}}} \tag{С.12}
\end{align*}
$$

$$
\begin{equation*}
\left.\frac{\text { energy }}{\text { distance }}\right|_{\text {ideal }}=\frac{P_{\text {total }}}{v}=\frac{1}{2} c_{\mathrm{d}} \rho A_{\mathrm{p}} v^{2}+\frac{1}{2} \frac{(m g)^{2}}{\rho v^{2} A_{\mathrm{s}}} \tag{C.13}
\end{equation*}
$$

$$
\frac{1}{2} c_{\mathrm{d}} \rho A_{\mathrm{p}} v^{2}
$$

$\frac{1}{2} \frac{(m g)^{2}}{\rho v^{2} A_{s}}$

$$
\frac{1}{2} c_{\mathrm{d}} \rho A_{\mathrm{p}} v^{2}
$$

$\frac{1}{2} \frac{(m g)^{2}}{\rho v^{2} A_{s}}$
$\rho=0.41$
$\epsilon=1 / 3$

$$
\begin{equation*}
\frac{\text { energy }}{\text { distance }}=\frac{1}{\epsilon}\left(\frac{1}{2} c_{\mathrm{d}} \rho A_{\mathrm{p}} v^{2}+\frac{1}{2} \frac{(m g)^{2}}{\rho v^{2} A_{\mathrm{s}}}\right) \tag{C.14}
\end{equation*}
$$

$$
\frac{1}{2} c_{\mathrm{d}} \rho A_{\mathrm{p}} v^{2}
$$

$$
\frac{1\left(\frac{(m)}{} \frac{2}{\rho c^{2} A_{s}}\right.}{}
$$

$$
\begin{equation*}
c_{\mathrm{d}} \rho A_{\mathrm{p}} v^{2}=\frac{(m g)^{2}}{\rho v^{2} A_{\mathrm{s}}}, \tag{C.15}
\end{equation*}
$$

$$
\begin{equation*}
\rho v_{\mathrm{opt}}^{2}=\frac{m g}{\sqrt{c_{\mathrm{d}} A_{\mathrm{p}} A_{\mathrm{s}}}} \tag{C.16}
\end{equation*}
$$

$$
\begin{align*}
\text { force } & =\left.\frac{\text { energy }}{\text { distance }}\right|_{\text {ideal }}=\frac{1}{2} c_{\mathrm{d}} \rho A_{\mathrm{p}} v^{2}+\frac{1}{2} \frac{(m g)^{2}}{\rho v^{2} A_{\mathrm{s}}}  \tag{С.17}\\
& =c_{\mathrm{d}} \rho A_{\mathrm{p}} v_{\mathrm{opt}}^{2}  \tag{C.18}\\
& =c_{\mathrm{d}} \rho A_{\mathrm{p}} \frac{m g}{\rho\left(c_{\mathrm{d}} A_{\mathrm{p}} A_{\mathrm{s}}\right)^{1 / 2}}  \tag{C.19}\\
& =\left(\frac{c_{\mathrm{d}} A_{\mathrm{p}}}{A_{\mathrm{s}}}\right)^{1 / 2} m g \tag{C.20}
\end{align*}
$$

$$
\begin{equation*}
f_{A}=\frac{A_{\mathrm{p}}}{A_{\mathrm{s}}} . \tag{C.21}
\end{equation*}
$$

$$
\begin{equation*}
\text { force }=\left(c_{\mathrm{d}} f_{A}\right)^{1 / 2}(m g) \tag{C.22}
\end{equation*}
$$

$c_{\mathrm{d}} \simeq 0.03$

$$
\begin{equation*}
\left(c_{\mathrm{d}} f_{A}\right)^{1 / 2} m g=0.036 m g=130 \mathrm{kN} . \tag{C.23}
\end{equation*}
$$

$$
\begin{align*}
\text { transport cost } & =\frac{1}{\epsilon} \frac{\text { force }}{\text { mass }}  \tag{C.24}\\
& =\frac{1}{\epsilon} \frac{\left(c_{\mathrm{d}} f_{A}\right)^{1 / 2} m g}{m}  \tag{C.25}\\
& =\frac{\left(c_{\mathrm{d}} f_{A}\right)^{1 / 2}}{\epsilon} g . \tag{C.26}
\end{align*}
$$

$\epsilon=1 / 3$
$0.4 \mathrm{kWh} /$ ton -km .

$$
c_{\mathrm{d}}
$$

$$
\begin{align*}
& \text { transport efficiency (passenger-km per litre of fuel) } \\
& \qquad \begin{array}{l}
\text { number of passengers } \times \frac{\text { energy per litre }}{\frac{\text { thrust }}{\epsilon}} \\
=\text { number of passengers } \times \frac{\epsilon \times \text { energy per litre }}{\text { thrust }} \\
=400 \times \frac{1}{3} \frac{38 \mathrm{MJ} / \text { litre }}{200000 \mathrm{~N}} \\
=25 \text { passenger }-\mathrm{km} \text { per litre }
\end{array} \tag{C.27}
\end{align*}
$$

$$
\begin{equation*}
\text { range }=v_{\text {opt }} \frac{\text { energy }}{\text { power }}=\frac{\text { energy } \times \epsilon}{\text { force }} \tag{С.31}
\end{equation*}
$$

$$
\begin{equation*}
\text { range }=\frac{\text { energy } \epsilon}{\text { force }}=\frac{C m \epsilon f_{\text {fuel }}}{\left(c_{\mathrm{d}} f_{A}\right)^{1 / 2}(m g)}=\frac{\epsilon f_{\text {fuel }}}{\left(c_{\mathrm{d}} f_{A}\right)^{1 / 2}} \frac{C}{g} \tag{C.32}
\end{equation*}
$$

$$
\left(\frac{\epsilon f_{\text {fuel }}}{\left(c_{\mathrm{d}} f_{A}\right)^{1 / 2}}\right)
$$

$\stackrel{c}{8}$

$$
\begin{equation*}
d_{\text {Fuel }}=\frac{C}{g}=4000 \mathrm{~km} . \tag{С.33}
\end{equation*}
$$

$$
\left(\frac{\epsilon f_{\text {fuel }}}{\left(c_{\mathrm{d}} f_{A}\right)^{1 / 2}}\right)
$$

$\epsilon=1 / 3$
$\left(c_{\mathrm{d}} f_{A}\right)^{1 / 2} \simeq 1 / 20$

$$
v^{2} \sim m g /\left(\rho\left(c_{\mathrm{d}} A_{\mathrm{p}} A_{\mathrm{s}}\right)^{1 / 2}\right)
$$

$0.4 \mathrm{kWh} /$ ton -km .
$0.4 \mathrm{kWh} /$ ton-km,
$m_{\text {total }}=\rho V$

$$
\begin{equation*}
F=\frac{1}{2} c_{d} A \rho v^{2}, \tag{C.35}
\end{equation*}
$$

$$
c_{\mathrm{d}}
$$

$$
\begin{align*}
\frac{F}{\epsilon m_{\text {total }}} & =\frac{\frac{1}{2} c_{\mathrm{d}} A \rho v^{2}}{\epsilon \rho \frac{2}{3} A L}  \tag{C.36}\\
& =\frac{3}{4 \epsilon} c_{\mathrm{d}} \frac{v^{2}}{L} \tag{C.37}
\end{align*}
$$

$$
\frac{F}{\epsilon m_{\text {total }}}=3 \times 0.03 \frac{(22 \mathrm{~m} / \mathrm{s})^{2}}{400 \mathrm{~m}}=0.1 \mathrm{~m} / \mathrm{s}^{2}=0.03 \mathrm{kWh} / \mathrm{t}-\mathrm{km}
$$

$\epsilon=1 / 3$

D Solar II

E Heating II
power loss $=$ area $\times U \times$ temperature difference.

$$
u_{\text {series combination }}=1 /\left(\frac{1}{u_{1}}+\frac{1}{u_{2}}\right)
$$

$$
\begin{align*}
\begin{array}{l}
\text { power } \\
\text { (watts) }
\end{array} & =C \frac{N}{1 \mathrm{~h}} V\left(\mathrm{~m}^{3}\right) \Delta T(\mathrm{~K})  \tag{E.1}\\
& =\left(1.2 \mathrm{~kJ} / \mathrm{m}^{3} / \mathrm{K}\right) \frac{N}{3600 \mathrm{~s}} V\left(\mathrm{~m}^{3}\right) \Delta T(\mathrm{~K})  \tag{E.2}\\
& =\frac{1}{3} N V \Delta T . \tag{E.3}
\end{align*}
$$

$$
\frac{1}{3} N V \times(\Delta T \times \text { duration }) .
$$

Something $\times(\Delta T \times$ duration $)$,
energy lost $=$ leakiness $\times$ temperature demand.
energy consumed $=$ energy delivered/coefficient of performance.
$322 \mathrm{~W} / \mathrm{m}^{2} /{ }^{\circ} \mathrm{C} \times 120$ degree-hours $\simeq 39 \mathrm{kWh}$.
$155 \mathrm{kWh} / \mathrm{d}$.
$7.7 \mathrm{kWh} / \mathrm{d} /{ }^{\circ} \mathrm{C} \times 2866$ degree-days $/ \mathrm{y} /(365$ days $/ \mathrm{y})=61 \mathrm{kWh} / \mathrm{d}$.

$$
1 /\left(\frac{1}{2.2}+\frac{1}{1.7}\right) \simeq 1 \mathrm{~W} / \mathrm{m}^{2} / \mathrm{K}
$$

$$
\text { efficiency }=\frac{T_{2}}{T_{2}-T_{1}}
$$

$$
\text { efficiency }=\frac{T_{2}}{T_{1}-T_{2}}
$$

$$
\begin{aligned}
\text { mass } & =\frac{\text { energy }}{\text { heat capacity } \times \text { temperature difference }} \\
& =\frac{24 \times 30 \times 3.6 \mathrm{MJ}}{\left(820 \mathrm{~J} / \mathrm{kg} /{ }^{\circ} \mathrm{C}\right)\left(50^{\circ} \mathrm{C}-16^{\circ} \mathrm{C}\right)} \\
& =100000 \mathrm{~kg},
\end{aligned}
$$

$$
\frac{1}{\sqrt{4 \pi \kappa t}} \exp \left(-\frac{x^{2}}{4(\kappa /(C \rho)) t}\right)
$$

$$
\sqrt{2 \frac{\kappa}{C_{\rho}}} t_{;}
$$

$\rho=2500$

$$
\rho=1000
$$

$$
5 \mathrm{~W} / \mathrm{m}^{2} \times 160 \mathrm{~m}^{2}=800 \mathrm{~W} / \mathrm{m}^{2}=19 \mathrm{kWh} / \mathrm{d} \text { per person. }
$$

$$
\text { Flux }=\kappa \times \frac{\Delta T}{h}=3 \mathrm{~W} / \mathrm{m}^{2}
$$

$$
\begin{equation*}
\frac{\partial T(z, t)}{\partial t}=\frac{\kappa}{C_{V}} \frac{\partial^{2} T(z, t)}{\partial z^{2}} \tag{E.4}
\end{equation*}
$$

$$
\begin{equation*}
T(0, t)=T_{\text {surface }}(t)=T_{\text {average }}+A \cos (\omega t), \tag{E.5}
\end{equation*}
$$

$$
\begin{equation*}
T(z, t)=T_{\text {average }}+A e^{-z / z_{0}} \cos \left(\omega t-z / z_{0}\right) \tag{E.6}
\end{equation*}
$$

$$
\begin{equation*}
z_{0}=\sqrt{\frac{2 \kappa}{C_{\mathrm{V}} \omega}} \tag{E.7}
\end{equation*}
$$

$$
\begin{equation*}
\kappa \frac{\partial T}{\partial z}=\kappa \frac{A}{z_{0}} \sqrt{2} e^{-z / z_{0}} \sin \left(\omega t-z / z_{0}-\pi / 4\right) \tag{E.8}
\end{equation*}
$$

$$
\begin{equation*}
\kappa \frac{A}{z_{0}} \sqrt{2}=A \sqrt{C_{\mathrm{V}} \kappa \omega} . \tag{E.9}
\end{equation*}
$$

F Waves II

$$
v=\frac{g T}{2 \pi}
$$

$\frac{1}{2} \rho h(\lambda / 2)$

$$
\begin{equation*}
P_{\text {potential }} \simeq \frac{1}{2} \rho h \frac{\lambda}{2} g h / T . \tag{F.2}
\end{equation*}
$$

$$
\begin{equation*}
P_{\text {potential }} \simeq \frac{1}{4} \rho g h^{2} v . \tag{F.3}
\end{equation*}
$$

$$
P_{\text {total }} \simeq \frac{1}{2} \rho g h^{2} v .
$$

$$
\begin{equation*}
P_{\text {total }}=\frac{1}{4} \rho g h^{2} v . \tag{F.5}
\end{equation*}
$$

$$
\begin{equation*}
P_{\text {total }}=\frac{1}{4} \rho g h^{2} v=40 \mathrm{~kW} / \mathrm{m} . \tag{F.6}
\end{equation*}
$$

G Tide II
$\rho \times(2 h)$

2phgh
banas
power per unit area of tide-pool $\simeq 3 \mathrm{~W} / \mathrm{m}^{2}$.

$$
\begin{equation*}
\rho g^{3 / 2} \sqrt{d} h^{2} / 2 . \tag{G.1}
\end{equation*}
$$

$\rho g^{3 / 2} \sqrt{d} h^{2} / 2$

$$
\begin{equation*}
v=\sqrt{g d} . \tag{G.2}
\end{equation*}
$$

$$
\begin{equation*}
K_{\mathrm{BV}}=\frac{1}{2} \rho A U^{3}, \tag{G.4}
\end{equation*}
$$

$$
\begin{equation*}
\text { power }=\frac{1}{2}\left(\rho g h^{2} \lambda\right) \times w / T=\frac{1}{2} \rho g h^{2} v \times w, \tag{G.6}
\end{equation*}
$$

$$
\begin{equation*}
\text { power }=\rho g h^{2} \sqrt{g d} \times w / 2=\rho g^{3 / 2} \sqrt{d} h^{2} \times w / 2 \tag{G.7}
\end{equation*}
$$

$$
\begin{equation*}
K_{\mathrm{BV}}=\frac{1}{2} \rho A U^{3}=\frac{1}{2} \rho w d(v h / d)^{3}=\rho\left(g^{3 / 2} / \sqrt{d}\right) h^{3} \times w / 2 . \tag{G.8}
\end{equation*}
$$

$$
\begin{equation*}
\frac{K_{\mathrm{BV}}}{\text { power }}=\frac{\rho w\left(g^{3 / 2} / \sqrt{d}\right) h^{3}}{\rho g^{3 / 2} h^{2} \sqrt{d} w}=\frac{h}{d} . \tag{G.9}
\end{equation*}
$$

$h \sim 1 / d^{1 / 4}$
$\frac{\text { power per tidemill }}{\text { area per tidemill }}=\frac{\pi}{200} \frac{1}{2} \rho U^{3}$.
$\epsilon_{\mathrm{g}}=0.9$

$$
\epsilon_{\mathrm{p}}=0.85
$$

$$
b / \epsilon_{\mathrm{p}}=\epsilon_{\mathrm{g}}(b+2 h) .
$$

$$
\epsilon=\epsilon_{\mathrm{g}} \epsilon_{\mathrm{p}}
$$

$$
b=2 h \frac{\epsilon}{1-\epsilon} .
$$

$\epsilon=76 \%$

$$
\left(\frac{1}{2} \rho g \epsilon_{\mathrm{g}}(b+2 h)^{2}-\frac{1}{2} \rho g \frac{1}{\epsilon_{\mathrm{p}}} b^{2}\right) / T,
$$

$\epsilon_{\mathrm{g}} 2 \rho g h^{2} / T$

$$
\left(\frac{1}{1-\epsilon}\right),
$$

## H Stuff II

## I Quick reference

$\simeq 0.37 \mathrm{~g}$

## J Populations and areas

K UK energy history

