

Tutorial Guide to C++ Programming

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This document provides an introduction to computing and the C++ programming language. It will help you teach yourself to write, compile, execute and test simple computer programs in C++ and describes some of the computing exercises to be completed in the IB course.

Please read the whole of this tutorial guide as soon as possible.

Pages 1–43 must be read *before* the first laboratory session.

Pages 32–48 should be read before the second laboratory session.

Pages 49–63 should be read before the third laboratory session.

The rest of the tutorial guide should be read before the second term.

For the latest information about the IB course, please see:

- the course wiki (which requires Raven access):

<https://wiki.csx.cam.ac.uk/cphysics/>

– also reachable via <http://tinyurl.com/32tw9m>

- the course homepage:

<http://www.inference.phy.cam.ac.uk/teaching/comput/C++/>

– also reachable via <http://tinyurl.com/2uffap>

You are advised to attend the laboratory sessions with expert demonstrators, which will take place in the PWF at the Cavendish Laboratory (Room 173, Bragg building, next to the 1A labs) at the following times in weeks 5, 6, 7, 8 of Michaelmas term and weeks 1–3 of Lent term.

	Fridays 2–4pm
	Mondays 2–4pm
Tuesdays 11am–1pm	Tuesdays 2–4pm
	Wednesdays 2–4pm

Bring your PWF password!

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A. Aims of the Computing Course

This guide provides a tutorial introduction to computing and the C++ computer programming language. It will help you teach yourself to **write, compile, execute and test simple computer programs** in C++ which read data from the keyboard, perform some computations on that data, and print out the results; and how to display the results graphically using `gnuplot`.

By the end of the course, you should be able to use computing to help you understand physics better.

C. Organisation

There are 4 two-hour laboratory sessions scheduled for the Michaelmas term computing course, and 3 sessions for the Lent term. You should work through the examples and exercises described in this document. Each session consists of a tutorial (which must be read **before** the start of the laboratory session) and interleaved computing exercises.

Please allow two hours preparation time per week.

The computing exercises are placed inside a box like this one.

To benefit most from the laboratory session and from the demonstrators' help you must **read the tutorial sections before the start of each session.**

C. Assessment

In this course, your progress will be **self-assessed**.

The self-assessment process is based on a desire to treat you as adults, and on the assumption that you are interested in learning, capable of choosing goals for yourself, and capable of self-evaluation. I expect everyone to achieve full marks.

Programming is an essential skill, just like **graph-sketching**. If you don't know how to sketch graphs, what do you do? – practice, read books, talk to supervisors and colleagues, practice some more. As second-year physicists, graph sketching should be second nature, and you should use it as a tool in all physics problems you study. Similarly, programming is a valuable tool for thinking, exploring, investigating, and understanding almost any scientific or technical topic. You will find that programming can help you with your 2nd year, 3rd year, and 4th year courses. This course exists to get you up to speed on programming, for the benefit of all your courses.

Many skills are best acquired gradually. Programming is one such skill, so this course is spread over two terms.

To ensure that you are making steady progress we will set a sequence of tasks and targets. We strongly encourage you to do these tasks at the recommended time, because that's when the expert demonstrators are there to help you. To allow for unexpected complications, the final deadline for assessment of each piece of work will be 9 days after the recommended completion time.

The main form of assessment for all this work will be self-assessment. Ask yourself “have I mastered this stuff?” If not, take appropriate action. Think, read, tinker, experiment, talk to demonstrators, talk to colleagues. Sort it out. Sort it out well before the final deadline. When you have checked your own work and fully satisfied your self-assessment, you should submit an electronic record of your self-assessment to the course pigeonholes. By submitting your work, you are confirming that you have assessed yourself and achieved the session’s objectives. Please do not abuse the electronic pigeonholes. I will make random checks, and all self-assessed submissions will be made available to the IB examiners.

Each session has tasks of two types: first, “write a program to do X”; and second “set your own additional goal (Y), further testing your skill, and write a program to do Y”. This “additional programming goal” doesn’t have to be any harder than X. You should choose the goal yourself. This goal and its solution are what you submit to the electronic course pigeonholes.

Collaboration

How you learn to program is up to you, but let me make a recommendation:

I recommend that you do most of your programming work in **pairs**, *with the weaker programmer doing the typing*, and the stronger one looking over his/her shoulder.

Working in pairs greatly reduces programming errors. Working in pairs means that there is always someone there to whom you can explain what you are doing. Explaining is a great way of enhancing understanding. It’s crucial to have the weaker programmer do the typing, so that both people in the pair understand everything that’s going on.

At the end of the day, you must all self-assess *individually*, and you must submit *individual* electronic records of your work to the electronic course pigeonholes.

Copying

When programming in real life, copying is strongly encouraged.

- Copying saves time;
- Copying avoids typing mistakes;
- Copying allows you to focus on your new programming challenges.

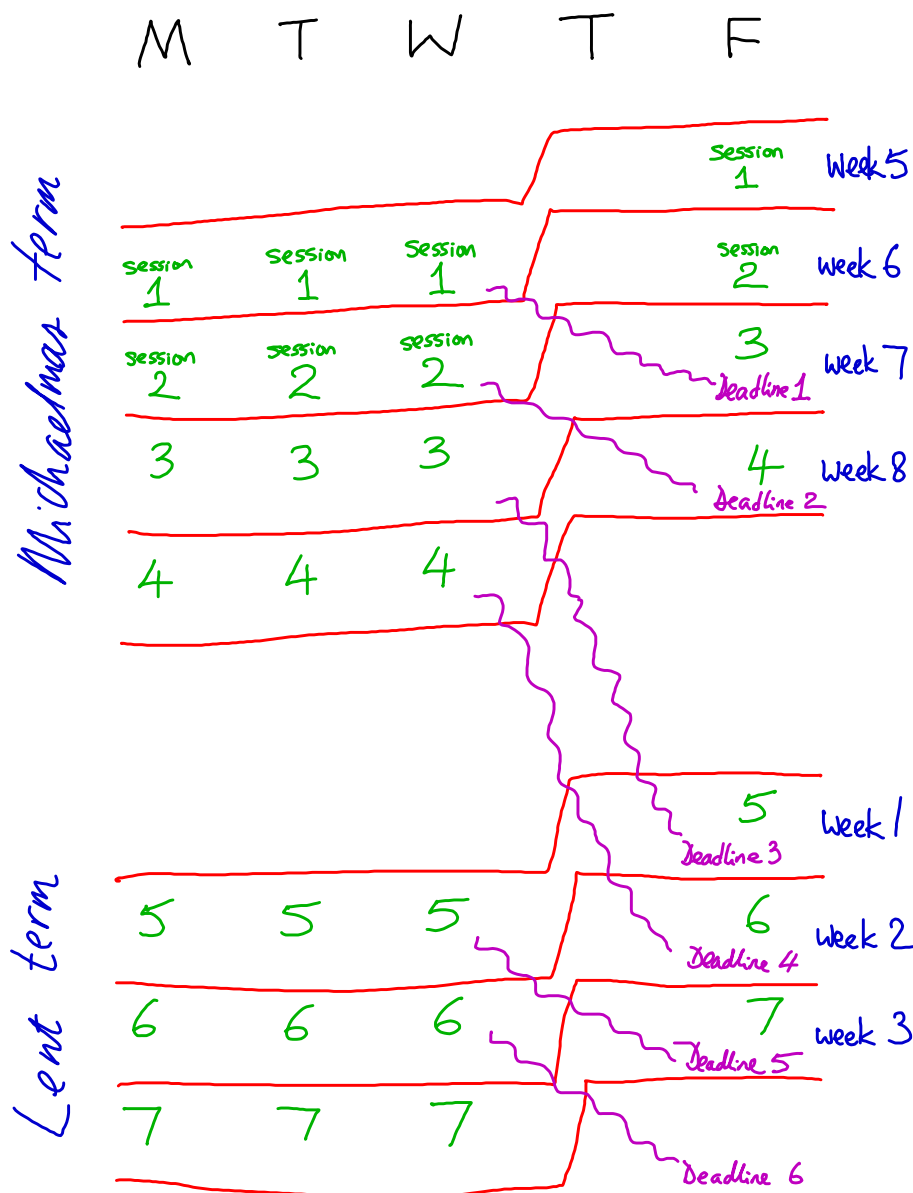
Similarly, in this course, copying may well be useful. For example, copy a working program similar to what you want to do; then modify it. Feel free to copy programs from the internet. The bottom line is: “do you understand how to solve this sort of programming problem?” Obviously, copying someone else’s perfect answer verbatim does *not* achieve the aim of learning to program. Always self-assess. If you don’t understand something you’ve copied, tinker with it until you do.

You should not copy anyone else’s “additional programming goal”. You should devise this goal yourself, and solve it yourself.

If necessary, set yourself further additional programming goals.

Deadlines

You should submit six pieces of self-assessed work. The diagram below explains the timetable. You should aim to complete "Session 1" during week 5, and submit your work that week. The final deadline for Session 1 work is nine days later, on the Friday of week 7. This pattern repeats for each of the six pieces of work.



Feedback

Your feedback on all aspects of this new course is welcome. Feedback given *early* is more valuable than feedback delivered through the end-of-term questionnaires. Early feedback allows any problems arising to be fixed immediately rather than next year!

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Twenty-nine useful unix commands

This guide, based on a University computing service leaflet, is intended to be as generally valid as possible, but there are many different versions of Unix available within the University, so if you find a command option behaving differently on your local machine you should consult the on-line manual page for that command. Some commands have numerous options and there is not enough space to detail them all here, so for fuller information on these commands use the relevant on-line manual page.

The names of commands are printed in **bold**, and the names of objects operated on by these commands (e.g. files, directories) are printed in *italics*.

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1. **cat** - display or concatenate files
2. **cd** - change directory
3. **chmod** - change the permissions on a file or directory
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5. **date** - display the current date and time
6. **diff** - display differences between text files
7. **file** - determine the type of a file
8. **find** - find files of a specified name or type
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- 21. **nice** - change the priority at which a job is being run
- 22. **passwd** - change your password
- 23. **ps** - list processes
- 24. **pwd** - display the name of your current directory
- 25. **quota** - disk quota and usage
- 26. **rm** - remove files or directories
- 27. **rmdir** - remove a directory
- 28. **sort** - sort and collate lines
- 29. **ssh** - secure remote login program

1. **cat** – display or concatenate files

cat takes a copy of a file and sends it to the standard output (i.e., to be displayed on your terminal, unless redirected elsewhere), so it is generally used either to read files, or to string together copies of several files, writing the output to a new file.

cat *ex*

displays the contents of the file *ex*.

cat *ex1 ex2 > newex*

creates a new file *newex* containing copies of *ex1* and *ex2*, with the contents of *ex2* following the contents of *ex1*.

2. **cd** – change directory

cd is used to change from one directory to another.

cd *dir1*

changes directory so that *dir1* is your new current directory. *dir1* may be either the full pathname of the directory, or its pathname relative to the current directory.

cd

changes directory to your home directory.

cd ..

moves to the parent directory of your current directory.

3. **chmod** – change the permissions on a file or directory

chmod alters the permissions on files and directories using either symbolic or octal numeric codes. The symbolic codes are given here:-

u	user	+	to add a permission	r	read
g	group	-	to remove a permission	w	write
o	other	=	to assign a permission explicitly	x	execute (for files), access (for directories)

The following examples illustrate how these codes are used.

chmod u=rw *file1*

sets the permissions on the file *file1* to give the user read and write permission on *file1*. No other permissions are altered.

chmod u+x,g+w,o-r *file2*

alters the permissions on the file *file2* to give the user execute permission on *file2*, to give members of the user's group write permission on the file, and prevent any users not in this group from reading it.

chmod u+w,go-x *dir1*

gives the user write permission in the directory *dir1*, and prevents all other users having access to that directory (by using **cd**. They can still list its contents using **ls**.)

chmod g+s *dir2*

means that files and subdirectories in *dir2* are created with the group-ID of the parent directory, not that of the current process.

4. **cp** – copy a file

The command **cp** is used to make copies of files and directories.

cp *file1 file2*

copies the contents of the file *file1* into a new file called *file2*. **cp** cannot copy a file onto itself.

cp *file3 file4 dir1*

creates copies of *file3* and *file4* (with the same names), within the directory *dir1*. *dir1* must already exist for the copying to succeed.

cp -r *dir2 dir3*

recursively copies the directory *dir2*, together with its contents and subdirectories, to the directory *dir3*. If *dir3* does not already exist, it is created by **cp**, and the contents and subdirectories of *dir2* are recreated within it. If *dir3* does exist, a subdirectory called *dir2* is created within it, containing a copy of all the contents of the original *dir2*.

5. **date** – display the current date and time

date returns information on the current date and time in the format shown below:-

```
Wed Jan 9 14:35:45 GMT 2002
```

6. **diff** – display differences between text files

diff *file1 file2* reports line-by-line differences between the text files *file1* and *file2*. The default output will contain lines such as ' *n1* a *n2,n3* ' and ' *n4,n5* c *n6,n7* ', (where ' *n1* a *n2,n3* ' means that *file2* has the extra lines *n2* to *n3* following the line that has the number *n1* in *file1*, and ' *n4,n5* c *n6,n7* ' means that lines *n4* to *n5* in *file1* differ from lines *n6* to *n7* in *file2*). After each such line, **diff** prints the relevant lines from the text files, with < in front of each line from *file1* and > in front of each line from *file2*.

There are several options to **diff**, including **diff -i**, which ignores the case of letters when comparing lines, and **diff -b**, which ignores all trailing blanks.

diff -cn produces a listing of differences within *n* lines of context, where the default is three lines. The form of the output is different from that given by **diff**, with + indicating lines that have been added, – indicating lines that have been removed, and ! indicating lines that have been changed.

diff *dir1 dir2* will sort the contents of directories *dir1* and *dir2* by name, and then run **diff** on the text files that differ.

7. **file** – determine the type of a file

file tests named files to determine the categories their contents belong to.

file *file1*

can tell if *file1* is, for example, a source program, an executable program or shell script, an empty file, a directory, or a library, but (a warning!) it does sometimes make mistakes.

8. **find** – find files of a specified name or type

find searches for files in a named directory and all its subdirectories.

find . -name '*.cc' -print

searches the current directory and all its subdirectories for files ending in *.cc*, and writes their names to the standard output. In some versions of Unix the names of the files will only be written out if the **-print** option is used.

find /local -name core -user user1 -print

searches the directory */local* and its subdirectories for files called *core* belonging to the user *user1* and writes their full file names to the standard output.

9. **ftp** – file transfer program

ftp is an interactive file transfer program. While logged on to one machine (described as the local machine), **ftp** is used to logon to another machine (described as the remote machine) that files are to be transferred to or from. As well as file transfers, it allows the inspection of directory contents on the remote machine. There are numerous options and commands associated with **ftp**, and **man ftp** will give details of those.

A simple example **ftp** session, in which the remote machine is hermes, is shown below:-

ftp *hermes.cam.ac.uk*

If the connection to hermes is made, it will respond with the prompt:-

Name (hermes.cam.ac.uk:user1) :

(supposing *user1* is your username on your local machine). If you have the same username on hermes, then just press *Return*; if it is different, enter your username on hermes before pressing *Return*. You will then be prompted for your hermes password, which will not be echoed.

After logging in using **ftp** you will be in your home directory on hermes. Some Unix commands, such as **cd**, **mkdir**, and **ls**, will be available. Other useful commands are:

help

lists the commands available to you while using **ftp**.

get *remote1 local1*

creates a copy on your local machine of the file *remote1* from hermes. On your local machine this new file will be called *local1*. If no name is specified for the file on the local machine, it will be given the same name as the file on hermes.

send *local2 remote2*

copies the file *local2* to the file *remote2* on hermes, i.e., it is the reverse of **get**.

quit

finishes the **ftp** session. **bye** and **close** can also be used to do this.

Some machines offer a service called “anonymous ftp”, usually to allow general access to certain archives. To use such a service, enter *anonymous* instead of your username when you ftp to the machine. It is fairly standard practice for the remote machine to ask you to give your email address as a password. Once you have logged on you will have read access in a limited set of directories, usually within the /pub directory tree. It is good etiquette to follow the guidelines laid down by the administrators of the remote machine, as they are being generous in allowing such access. See leaflet G72 for more detailed examples of using ftp.

WARNING! When you use **ftp** the communications between the machines are not encrypted. This means that your password could be snooped when you use it make an **ftp** connection. If available, the commands **sftp** (secure file transfer program) or **scp** (secure remote file copy program) are preferable, as they provide encrypted file transfer. Use **man scp** to get the syntax, which is like **cp**’s. To copy file1 from a remote machine to your local machine and name it file2:

```
scp user@pwf.cam.ac.uk:file1 file2
```

To copy file2 from here to your home directory on a remote machine:

```
scp file2 user@pwf.cam.ac.uk:~
```

10. **grep** – searches files for a specified string or expression

grep searches for lines containing a specified pattern and, by default, writes them to the standard output.

```
grep motif1 file1
```

searches the file *file1* for lines containing the pattern *motif1*. If no file name is given, **grep** acts on the standard input. **grep** can also be used to search a string of files, so

```
grep motif1 file1 file2 ... fileN
```

will search the files *file1*, *file2*, ... , *fileN*, for the pattern *motif1*.

```
grep -c motif1 file1
```

will give the number of lines containing *motif1* instead of the lines themselves.

```
grep -v motif1 file1
```

will write out the lines of *file1* that do NOT contain *motif1*.

11. **gzip** – compress a file

gzip reduces the size of named files, replacing them with files of the same name extended by .gz. The amount of space saved by compression varies.

```
gzip file1
```

results in a compressed file called *file1.gz*, and deletes *file1*.

```
gzip -v file2
```

compresses *file2* and gives information, in the format shown below, on the percentage of the file’s size that has been saved by compression:-

```
file2 : Compression 50.26% -- replaced with file2.gz
```

To restore files to their original state use the command **gunzip**. If you have a compressed file *file2.gz*, then

gunzip file2

will replace *file2.gz* with the uncompressed file *file2*.

12. **help** – display info about bash builtin commands

help gives access to information about builtin commands in the **bash** shell. Using **help** on its own will give a list of the commands it has information about. **help** followed by the name of one of these commands will give information about that command. **help history**, for example, will give details about the **bash** shell history listings.

13. **info** – read online documentation

info is a hypertext information system. Using the command **info** on its own will enter the **info** system, and give a list of the major subjects it has information about. Use the command **q** to exit **info**. For example, **info bash** will give details about the **bash** shell.

14. **kill** – kill a process

To kill a process using **kill** requires the process id (PID). This can be found by using **ps**. Suppose the PID is 3429, then

kill 3429

should kill the process. If it doesn't then sometimes adding **-9** helps...

kill -9 3429

15. **lpr** – print out a file

lpr is used to send the contents of a file to a printer. If the printer is a laserwriter, and the file contains PostScript, then the PostScript will be interpreted and the results of that printed out.

lpr -Pprinter1 file1

will send the file *file1* to be printed out on the printer *printer1*. To see the status of the job on the printer queue use

lpq -Pprinter1

for a list of the jobs queued for printing on *printer1*. (This may not work for remote printers.)

16. **ls** – list names of files in a directory

ls lists the contents of a directory, and can be used to obtain information on the files and directories within it.

ls *dir1*

lists the names of the files and directories in the directory *dir1*, (excluding files whose names begin with **.**). If no directory is named, **ls** lists the contents of the current directory.

ls -a *dir1*

will list the contents of *dir1*, (including files whose names begin with **.**).

ls -l *file1*

gives details of the access permissions for the file *file1*, its size in kbytes, and the time it was last altered.

ls -l *dir1*

gives such information on the contents of the directory *dir1*. To obtain the information on *dir1* itself, rather than its contents, use

ls -ld *dir1*

17. **man** – display an on-line manual page

man displays on-line reference manual pages.

man *command1*

will display the manual page for *command1*, e.g **man** *cp*, **man** *man*.

man -k *keyword*

lists the manual page subjects that have *keyword* in their headings. This is useful if you do not yet know the name of a command you are seeking information about, but can produce a lot of output. To refine the output you could, for example, use **man -k** *keyword* | **grep** '**1**' to get a list of user commands with *keyword* in their headings (user commands are in section 1 of the man pages). The | means that the output of **man -k** is piped to (i.e., is used as the input for) **grep**.

man -M*path* *command1*

is used to change the set of directories that **man** searches for manual pages on *command1*

18. **mkdir** – make a directory

mkdir is used to create new directories. In order to do this you must have write permission in the parent directory of the new directory.

mkdir *newdir*

will make a new directory called *newdir*.

mkdir -p can be used to create a new directory, together with any parent directories required.

mkdir -p *dir1/dir2/newdir*

will create *newdir* and its parent directories *dir1* and *dir2*, if these do not already exist.

19. **more** – scan through a text file page by page

more displays the contents of a file on a terminal one screenful at a time.

more *file1*

starts by displaying the beginning of *file1*. It will scroll up one line every time the *return* key is pressed, and one screenful every time the *space* bar is pressed. Type **?** for details of the commands available within **more**. Type **q** if you wish to quit **more** before the end of *file1* is reached.

more -n *file1*

will cause *n* lines of *file1* to be displayed in each screenful instead of the default (which is two lines less than the number of lines that will fit into the terminal's screen).

20. **mv** – move or rename files or directories

mv is used to change the name of files or directories, or to move them into other directories. **mv** cannot move directories from one file-system to another, so, if it is necessary to do that, use **cp** instead – copy the whole directory using **cp -r oldplace newplace** then remove the old one using **rm -r oldplace**.

mv *name1 name2*

changes the name of a file called *name1* to *name2*.

mv *dir1 dir2*

changes the name of a directory called *dir1* to *dir2*, unless *dir2* already exists, in which case *dir1* will be moved into *dir2*.

mv *file1 file2 dir3*

moves the files *file1* and *file2* into the directory *dir3*.

21. **nice** – change the priority at which a job is being run

nice causes a command to be run at a lower than usual priority. **nice** can be particularly useful when running a long program that could cause annoyance if it slowed down the execution of other users' commands. An example of the use of **nice** is

nice gzip *file1*

which will execute the compression of *file1* at a lower priority.

If the job you are running is likely to take a significant time, you may wish to run it in the background, i.e., in a subshell. To do this, put an ampersand **&**, after the name of your command or script. For instance,

rm -r mydir &

is a background job that will remove the directory *mydir* and all its contents.

The command **jobs** gives details of the status of background processes, and the command **fg** can be used to bring such a process into the foreground.

22. **passwd** – change your password

Use **passwd** when you wish to change your password. You will be prompted once for your current password, and twice for your new password. Neither password will be displayed on the screen.

23. **ps** – list processes

ps displays information on processes currently running on your machine. This information includes the process id, the controlling terminal (if there is one), the cpu time used so far, and the name of the command being run.

ps

gives brief details of your own processes in your current session.

To obtain full details of all your processes, including those from previous sessions use:-

ps -fu user1

using your own user name in place of *user1*.

ps is a command whose options vary considerably in different versions of Unix. Use **man ps** for details of all the options available on the machine you are using.

24. **pwd** – display the name of your current directory

The command **pwd** gives the full pathname of your current directory.

25. **quota** – display disk quota and usage

quota gives information on a user's disk space quota and usage.

On some systems using **quota** without options will only give details of where you have exceeded your disk quota on local disks, in which case, use the -v option

quota -v

to get details of your quota and usage on all mounted filesystems.

26. **rm** – remove files or directories

rm is used to remove files. In order to remove a file you must have write permission in its directory, but it is not necessary to have read or write permission on the file itself.

rm file1

will delete the file *file1*. If you use

rm -i file1

instead, you will be asked if you wish to delete *file1*, and the file will not be deleted unless you answer **y**. This is a useful safety check when deleting lots of files.

rm -r dir1

recursively deletes the contents of *dir1*, its subdirectories, and *dir1* itself, and should be used with suitable caution.

rm -rf dir1

is like **rm -r**, except that any write-protected files in the directory are deleted without query. This should be used with even more caution.

27. **rmdir** – remove a directory

rmdir removes named empty directories. If you need to delete a non-empty directory **rm -r** can be used instead.

rmdir *exdir*

will remove the empty directory *exdir*.

28. **sort** – sort and collate lines

The command **sort** sorts and collates lines in files, sending the results to the standard output. If no file names are given, **sort** acts on the standard input. By default, **sort** sorts lines using a character by character comparison, working from left to right, and using the order of the standard character set.

sort -d

uses “dictionary order”, in which only letters, digits, and white-space characters are considered in the comparisons.

sort -r

reverses the order of the collating sequence.

sort -n

sorts lines according to the arithmetic value of leading numeric strings. Leading blanks are ignored when this option is used, (except in some System V versions of **sort**, which treat leading blanks as significant. To be certain of ignoring leading blanks use **sort -bn** instead.).

29. **ssh** – secure remote login program

ssh is used for logging onto a remote machine, and provides secure encrypted communications between the local and remote machines using the SSH protocol. The remote machine must be running an SSH server for such connections to be possible. For example,

ssh -X *linux.phy.pwf.cam.ac.uk*

will commence a login connection to the Physics PWF server.

You can connect using your password for the remote machine, or you can set up a system of passphrases to avoid typing login passwords directly (see the man page for **ssh-keygen** for information on how to create these).

If you have a different user-id on the remote machine, use

ssh -X *YourUserID@linux.phy.pwf.cam.ac.uk*

The optional -X flag makes it possible for the remote machine to open X windows on your local machine.

1 Introduction

1.1 What is a computer program?

Computers process data, perform computations, make decisions and instigate actions under the control of sets of instructions called **computer programs**. The computer (central processing unit, keyboard, screen, memory, disc) is commonly referred to as the **hardware** while the programs that run on the computer (including operating systems, word processors and spreadsheets) are referred to as **software**.

Program instructions are stored and processed by the computer as a sequence of binary digits (i.e., 1s and 0s) called *machine code*. In the early days programmers wrote their instructions in strings of numbers called **machine language**. Although these machine instructions could be directly read and executed by the computer, they were too cumbersome for humans to read and write. Later, *assemblers* were developed to map machine instructions to English-like abbreviations called mnemonics (or **assembly language**). In time, **high-level programming languages** (e.g. FORTRAN (1954), COBOL (1959), BASIC (1963) and PASCAL (1971)) developed, which enable people to work with something closer to the words and sentences of everyday language. The instructions written in the high-level language are automatically translated by a **compiler** (which is just another program) into machine instructions which can be executed by the computer later. The compiled program may be called a 'binary' or an 'executable'.

Note that the word "program" is used to describe both the set of written instructions created by the programmer and also to describe the resulting piece of executable software.

1.2 The C++ Programming Language

The C++ programming language (Stroustrup (1988)) evolved from C (Ritchie (1972)) and is emerging as the standard in software development. For example, the *Unix* and *Windows* operating systems and applications are written in C and C++. C++ facilitates a structured and disciplined approach to computer programming called *object-oriented programming*. This course will cover the basic elements of C++.

2 Editing, compiling and executing a simple program

2.1 A simple C++ program to add two numbers

The following is an example of a simple program (source code) written in the C++ programming language. The program is short but nevertheless complete. The program is designed to read two numbers typed by a user at the keyboard, compute their sum and print the result on the screen.

```

// Program to add two integers typed by user at keyboard
#include <iostream>
using namespace std;

int main()
{
    int a, b, total;

    cout << "Enter integers to be added:" << endl;
    cin >> a >> b;
    total = a + b;
    cout << "The sum is " << total << endl;

    return 0;
}

```

2.2 Overview of program structure and syntax

C++ uses notation that may appear strange to non-programmers. The notation is part of the **syntax** of a programming language, i.e., the formal rules that specify the structure of a legal program. These rules will be explained in more detail later, in sections 3–5. Here we give a quick overview.

Every C++ program consists of a header and a main body and has the following structure.

```

// Comment statements, which are ignored by computer but inform reader
#include <header file name>

int main()
{
    declaration of variables;
    statements;

    return 0;
}

```

We will consider each line of the program: for convenience the program is reproduced on the next page with line numbers to allow us to comment on details of the program. You must **not** put line numbers in an actual program.

```
1 // Program to add two integers typed by user at keyboard
2 #include <iostream>
3 using namespace std;
4
5 int main()
6 {
7     int a, b, total;
8
9     cout << "Enter integers to be added:" << endl;
10    cin >> a >> b;
11    total = a + b;
12    cout << "The sum is " << total << endl;
13
14    return 0;
15 }
```

- Line 1** At the top of the program are comments and instructions, which will not be executed. Lines beginning with `//` indicate that the rest of the line is a **comment**. Comments are inserted by programmers to help people read and understand the program. Here the comment states the purpose of the program. Comments are not executed by the computer. They can in general be placed anywhere in a program.
- Line 2** Lines beginning with `#` are instructions to the compiler's *preprocessor*. The **include** instruction says "what follows is a file name, find that file and insert its contents right here". It is used to include the contents of a file of definitions that may be used in the program. Here the file `iostream` contains the definitions of some of the symbols used later in the program (e.g. **cin**, **cout**).
- Line 3** This is used to specify that names used in the program (such as **cin** and **cout**) are defined in the standard C and C++ libraries. This is used to avoid problems with other libraries that may also use these names.
- Line 5** When this program is executed the instructions will be executed in the order they appear in the main body of the program. The main body is delimited by `main()` and the opening and closing braces (curly brackets). This line also specifies that `main()` will return a value of type integer (`int`) on its completion (see line 14). Every C++ program, irrespective of what it is computing, begins in this way.
- Line 6** The opening (left) brace marks the beginning of the main body of the program. The main body consists of instructions, which are **declarations** defining the data or **statements** on how the data should be processed. All C++ declarations and statements must end with a semicolon.
- Line 7** This is a **declaration of variables** – like the cast of characters that precedes the script of a play. The words `a`, `b` and `total` are the names of **variables**. A variable is a location in the computer's memory where a value can be stored for use by a program. We can assign and refer to values stored at these locations by simply using the variable's name. The declaration also specifies the variables' **type**. Here the variables `a`, `b` and `total` are

declared to be data of type **int**, which means these variables hold integer values. At this stage, the variables' values are undefined.

Line 9 This statement instructs the computer to output the *string* of characters contained between the quotation marks, followed by a new line (`endl`). The location of the output is denoted by `cout`, which in this case will be the terminal screen.

Line 10 This statement tells the computer to read data typed in at the keyboard (standard input), denoted by `cin`. These values are *assigned to* (stored in) variables `a` and `b`.

Line 11 This statement is an **arithmetic expression** assigning the value of the expression `a + b` (the sum of the integer values stored at `a` and `b`) to the variable `total`.

Line 12 Instructs the computer to display the value of the variable `total`.

Line 14 The last instruction of every program is the return statement. The return statement with the integer value 0 (zero) is used to indicate to the operating system that the program has terminated successfully.

Line 15 The closing (right) brace marks the end of the main body of the program.

Blank lines (Lines 4, 8 and 13) have been introduced to make the program more readable. They will be ignored by the compiler. **Whitespace** (spaces, tabs and newlines) are also ignored (unless they are part of a string of characters contained between quotation marks). They too can enhance the visual appearance of a program.

Indentation It does not matter where you place statements, either on the same line or on separate lines. A common and accepted *style* is that you indent after each opening brace and move back at each closing brace.

2.3 The development environment and the development cycle

C++ programs go through 3 main phases during development: editing (writing the program), compiling (i.e., translating the program to executable code and detecting syntax errors) and running the program and checking for logical errors (called *debugging*).

1. Edit

The first phase consists of editing a file by typing in the C++ program with a text editor and making corrections if necessary. The program is stored as a text file on the disk, usually with the file extension `.cc` to indicate that it is a C++ program (e.g. `SimpleAdder.cc`).

2. Compile

A compiler translates the C++ program into machine language code (**object code**), which it stores on the disk as a file with the extension `.o` (e.g. `SimpleAdder.o`). A linker then links the object code with standard library routines that the program may use and creates an **executable image**, which is also saved on disk, usually as a file with the file name without any extension (e.g. `SimpleAdder`).

3. Execute

The executable is loaded from the disk to memory and the computer's processing unit (Central Processing Unit) executes the program one instruction at a time.

A. Objectives

- Familiarisation with the teaching system and C++ development environment
- Edit, compile and execute a working program
- Become familiar with the compiler's messages

B. Getting started

If you have not already done so, read sections 1–2 of the tutorial guide and then log onto a computer in the PWF by typing in your user identifier (the beginning of your e-mail address) and your password. We recommend using the PWF machines as linux machines; this may require a reboot. Once you have logged in, it might be a good idea to **open a web browser** and check if there's anything new on the course wiki:

`https://wiki.csx.cam.ac.uk/cphysics/`

(also reachable via `http://tinyurl.com/32tw9m`).

We recommend trying an old-fashioned style of computer programming, working from a terminal window. This style of working has the advantages that it can be done remotely over low-bandwidth connections, and it blends naturally into automation (in contrast to pointing and clicking, which tends to keep a cumbersome human in the loop). To **open a terminal**, you can right-click on your desktop and select 'Open Terminal'; there is a keyboard shortcut for this action: 'menu-T'. (The menu key is a little to the right of the space key.)

Once your terminal has opened you can run unix commands. Open your favourite plain text editor. To open emacs, you can type

```
emacs &
```

in a terminal. (Just typing emacs would also work, but then your terminal would remain locked up waiting for the emacs process to finish. Putting the & at the end of the unix command makes the emacs process run 'in the background'.)

C. Computing exercise

Use an editor to type in the simple adder program (SimpleAdder.cc) listed on page 18 and then compile and run it.

Execute it with different numbers at the input and check that it adds up the numbers correctly. What happens if you input two numbers with decimal places?

D. Editing, compiling and executing the simple adder program

1. Create a new folder

You'll be creating many files. To keep things tidy we suggest that you get into the habit of creating a folder or sub-folder for each topic you tackle. So start as you mean to go on – create a new Folder called, say, `c++`. You can do this in a terminal with the command

```
mkdir c++
```

2. Create and edit a new file

Create an Empty File called `SimpleAdder.cc` – the `.cc` extension informs the editor that the file will be a C++ program. Avoid putting spaces into the file name. (If you really want to avoid typing, you can find the file in `/ux/physics/part_1b/c++/examples/`.)

One bit of emacs jargon: when you open or load a file, emacs calls it a “buffer”.

Emacs understands enough about C++ to be able to colour-code the text (comments in one colour, keywords in another, etc). It also tries to indent your code in the approved style. To ask Emacs to reindent a line you've edited, press the Tab key.

3. Compile

To compile your program, get your terminal into the right directory (that is, the same folder where the `.cc` file is located) – type `cd c++`. Make sure you've saved the file, and type

```
g++ SimpleAdder.cc -o SimpleAdder
```

If all is well, an executable called `SimpleAdder` will have been created in your folder. (Type `ls` to check.) If you have made a typing mistake, find the error and correct it using the editor. Save the corrected file and try the compilation again.

4. Run

You can execute the program by typing `./SimpleAdder` in a terminal. When the program prompt appears, input the two integer numbers to the program by typing in two numbers separated by spaces and followed by a return. The program should respond by printing out the sum of the two numbers.

You can run the program as many times as you wish.

5. Modify

Modify the program in your editor to change its behaviour in some way. Recompile the program. It's a bit of a faff typing the whole command

```
g++ SimpleAdder.cc -o SimpleAdder
```

isn't it? Try typing

```
make SimpleAdder
```

instead. 'make' is a smart unix program that knows how to do things. If you tell it to make `SimpleAdder` and it notices that there's a file called `SimpleAdder.cc` nearby, make guesses the right thing to do.

E. Exploring compiler error messages

As you write computer programs in C++, you will repeatedly edit, compile, and run programs. And sometimes the compiler will give you error messages. Often the messages can be quite cryptic. It's a good idea to make some deliberate errors, to get a feel for what your future errors might mean. (Just as trainee doctors learn best by giving known diseases to patients.)

Grab a copy of the program `/ux/physics/part_1b/c++/examples/HelloFor.cc` listed below. (Also available from <http://tinyurl.com/2g4bo8>.)

```
// HelloFor.cc

#include <iostream>
using namespace std;

int main()
{
    for (int i=1; i<=10; i++) {
        cout << i << " hello world" << endl ;
        if ( i == 7 ) {
            cout << "that was lucky!" << endl ;
        } else {
            cout << endl ;
        }
    }
}
```

You can copy it to your current directory (known as `'.'`) with the unix command:

```
cp /ux/physics/part_1b/c++/examples/HelloFor.cc .
```

First compile this program and run it. Remember, once you've got the `.cc` file, you can compile it with the command `make HelloFor`; run it with the command `./HelloFor`

This program contains several C++ features that you haven't learned yet, but don't worry; take a look at the program and the output, and see if you can figure out roughly what's going on in the lines following the word `main`. The program uses the special C++ commands `for` and `if`.

Once you have a feel for what the program is doing, modify the program and explore the error-messages generated by the compiler when you compile programs with deliberate syntax errors.

For example, try omitting a quote character (`"`), or inserting an extra quote character. Try omitting the semicolon (`;`) at the end of a line. Try omitting or adding a left-brace (`{`) or right-brace (`}`). How many different types of error can you get the compiler to produce? Make a list. When the compiler gives error messages, it tells you the line-number where it finds an error. Is this line-number always the same as the line number where you introduced the typing error?

Assuming you make a single-character typing error, how big can you make the distance between the actual line-number of the error and the line-number where the compiler reports the error?

This knowledge of deliberately inflicted diseases and their symptoms should help you diagnose future bugs.

F. Compiler warnings

What happens if you replace the equality test (`i==7`) by (`i=7`)? Can you guess why?

[By the way: if you want to stop a running program, hit ctrl-C.]

Now try compiling the program with the incorrect equals sign using this command:

```
g++ -Wall HelloFor.cc -o HelloFor
```

The flag `-Wall` means 'Warnings (all)'. What difference does adding the flag `-Wall` make?

Suggested lesson: always compile with Warnings switched on.

G. Makefiles

You can switch the warning flags on automatically by telling make your own personal rules. Rules for make are normally placed in a file called `makefile`. You can grab an example like this:

```
cp /ux/physics/part_1b/c++/examples/makefile .
```

This `makefile` tells make to use `-Wall` and a few other flags when compiling C++ programs. When you've copied the `makefile` to the directory where `HelloFor.cc` is located, you can type `make HelloFor` again and see what happens for the two versions of the program. Notice that make only actually runs the compiler if the source file `HelloFor.cc` has been touched and thus needs recompiling.

H. Practice

The example `SimpleAdder.cc` showed you how to make a simple interactive program, which prompts the user for an integer using the special command `cin`.

Modify `HelloFor.cc` using code from `SimpleAdder.cc` to make a program that first asks the user to enter a lucky number (which plays the role of 7), then prints out the numbers 1 to 10, highlighting the lucky number in some way when it is printed. Modify the program so that the number of loops (currently 10) is also chosen interactively by the user.

I. Assessment

This course proceeds by self-assessment. Here is the part of session 1 that you must submit to the course pigeonhole. You should do this bit by yourself. It's fine to do all the other work in pairs, as long as it is the weaker programmer who does the typing; but this bit, the assessed bit, you must do alone.

Set yourself an **additional programming task** along the lines of the preceding tasks, and solve it.

When you are done, write a file called `README` or `README.txt` like this:

I set myself the following additional goal (devised by me):

.

My solution is in the file `_____.cc`

If you are confident that you have mastered the material in Session 1, pat yourself on the back, and submit your work to the course pigeonholes (as described below). Otherwise, seek help right away. Seek help from fellow students, from teachers, or from the course wiki.

When you have set yourself your additional programming task and solved it, you must submit your work to the course pigeonholes. Here's how that works.

Instructions for submitting your self-assessed work

The course pigeonhole is electronic, and is contained in the folder `/ux/physics/part_1b/c++/pigeonhole/` on the PWF.

1. The first time you submit work to the pigeonhole, make a folder therein named with your user ID: for example, if your user ID is `jb007`:

```
mkdir /ux/physics/part_1b/c++/pigeonhole/jb007
```

You may find it convenient to make a symbolic link (or 'shortcut') to your folder, like this for example

```
ln -s /ux/physics/part_1b/c++/pigeonhole/jb007 ~/submissions
```

2. Then, each time you submit a new piece of work, make a folder for it, with the name that identifies the task. For example, let's call the first task in the IB course "IB.1". Make a folder like this:

```
mkdir /ux/physics/part_1b/c++/pigeonhole/jb007/IB.1
```

[or `mkdir ~/submissions/IB.1` if you made the symbolic link].

3. Finally, put your work (the additional programming goal, and your solution) in that sub-folder: for example, if the work is in files called `c++/README` and `solution.cc`, copy them like this:

```
cp c++/README /ux/physics/part_1b/c++/pigeonhole/jb007/IB.1
cp solution.cc /ux/physics/part_1b/c++/pigeonhole/jb007/IB.1
```

Your marks for the course depend on your submission of your self-assessments to the course pigeonhole.

Deadline: You are strongly encouraged to complete this work on time during week 5. See page 5 for the final deadlines.

3 Variables and constants

Programs need a way to store the data they use. Variables and constants offer various ways to represent and manipulate data. Constants, as the name suggests, have fixed values. Variables, on the other hand, hold values that can be assigned and changed as the program executes.

3.1 Variable types

Every variable and constant has an associated **type**, which defines the set of values that can be legally stored in it. Variables can be conveniently divided into integer, floating point, character and boolean types for representing integer (whole) numbers, floating point numbers (real numbers with a decimal point), the ASCII character set (for example 'a', 'b', 'A') and the boolean set (true or false) respectively.

More complicated types of variable can be defined by a programmer, but for the moment, we will deal with just the simple C++ types. These are listed below:

int	to store a positive or negative integer (whole) number
float	to store a real (floating point) number
bool	to store the logical values true or false
char	to store one of 256 character (text) values

3.2 Declaration of a variable

A variable is introduced into a program by a declaration, which states the variable's **type** (i.e., int, float, bool, or char) and its name, which you are free to choose. A **declaration** must take the form:

<i>type</i>	<i>variable-name</i> ;
int	count;
float	length;
char	firstInitial;
bool	switched_on;

or:

<i>type</i>	<i>variable1, variable2, ... variableN</i> ;
float	base, height, areaCircle;
int	myAge, number_throws;

The variable name can be any sequence of characters consisting of letters, digits and underscores that do not begin with a digit. It must not be a special keyword of the C++ language and cannot contain spaces. C++ is case-sensitive: uppercase and lowercase letters are considered to be different. Good variable names tell you how the variable is used and help you understand the flow of the program. If you want to use a name made of two or more words, you may indicate the word boundaries by the underscore symbol (`_`) or by using an uppercase letter for the beginning of words.

3.3 Storage of variables in computer memory

When you run your program it is loaded into computer memory (RAM) from the disk file. A variable is in fact a location in the computer's memory in which a value can be stored and later retrieved. The variable's name is merely a label for that location – a *memory address*. It may help to think of variables as named boxes into which values can be stored and retrieved.

The amount of memory required for the variables depends on their type. This can vary between machines and systems but is usually one byte (8 bits) for a char variable, four bytes (32 bits) for an int and four bytes for a float. This imposes limits on the range of numbers assigned to each variable. Integer numbers must have values in the range -2147483648 to 2147483647 (i.e., $\pm 2^{31}$). Floats must be real numbers with magnitudes in the range 5.9×10^{-39} to 3.4×10^{38} (i.e., 2^{-127} to 2^{128}). They are usually stored using 1 bit for the sign (s), 8 bits for the exponent (E) and 23 bits for the mantissa (m) such that the number is equal to $s \times m \times 2^E$. The number of bits in the mantissa determines the accuracy of addition and subtraction. If two numbers are added and the smaller is smaller than $2^{-23} \approx 10^{-7}$ times the larger then the outcome of the addition will simply be the larger number. This inaccuracy of addition can cause problems for numerical methods such as dynamical simulations.

If an application requires very small or large numbers beyond these ranges, or greater precision, C++ offers an additional data types for floating point numbers: double. Variables of type double require double the amount of memory for storage but are useful when computing values to a high precision. Some programmers follow the rule 'always use doubles, not floats'.

4 Assignment of variables

4.1 Assignment statements

It is essential that every variable in a program is given a value explicitly before any attempt is made to use it. It is also important that the value assigned is of the correct type.

The most common form of statement in a program uses the **assignment operator**, =, and either an **expression** or a **constant** to assign a value to a variable:

```
variable = expression;  
variable = constant;
```

The symbol of the assignment operator looks like the mathematical equality operator **but** in C++ its meaning is different. The assignment statement indicates that the value given by the expression on the right hand side of the **assignment operator** (symbol =) must be stored in the variable named on the left hand side. The assignment operator should be read as "becomes equal to" and means that the **variable** on the left hand side has its value changed to the value of the **expression** on the right hand side. (In other programming languages the assignment operator is represented by := or <=.) For the assignment to work successfully, the type of the variable on the left hand side should be the same as the type returned by the expression.

The statement in line 11 of the simple adder program is an example of an assignment statement involving an **arithmetic expression**.

```
total = a + b;
```

It takes the values of a and b, sums them together and assigns the result to the variable total. As discussed above, variables can be thought of as named boxes into which values can be

stored. Whenever the name of a box (i.e., a variable) appears in an expression, it represents the value currently stored in that box. When an assignment statement is executed, a new value is dropped into the box, replacing the old one. Thus, line 10 of the program means “get the value stored in the box named `a`, add it to the value stored in the box named `b` and store the result in the box named `total`.”

The assignment statement:

```
total = total + 5;
```

is thus a valid statement since the new value of `total` becomes the old value of `total` with 5 added to it. Remember the assignment operator (`=`) is **not** the same as the equality operator in mathematics (represented in C++ by the operator `==`).

4.2 Arithmetic expressions

Expressions can be constructed out of variables, constants, operators and brackets. The commonly used mathematical or **arithmetic operators** include:

operator	operation
+	addition
-	subtraction
*	multiplication
/	division
%	modulus (modulo division)

The definitions of the first four operators are as expected. The modulo division (modulus) operation with an integer is the remainder after division, e.g. $13 \bmod 4$ (`13%4`) gives the result 1.

Although addition, subtraction and multiplication are the same for both integers and reals (floating point numbers), division is different. If you write (see later for declaration and initialisation of variables on the same line):

```
float a=13.0, b=4.0, result;  
result = a/b;
```

then a real division is performed and 3.25 is assigned to `result`. A different result would have been obtained if the variables had been defined as integers:

```
int i=13,j=4, result;  
result = i/j;
```

here `result` is assigned the integer value 3.

The remainder after integer division can be determined by the modulo division (modulus) operator, `%`. For example, the value of `i%j` would be 1. Be careful when using division and modulo division with negative integers. What is $-13/4$? What is $-13\%4$? It's easy to get tripped up because there are two possible conventions, and each programming language makes its own choice. In most languages the division and remainder operators will be *consistent* in the sense that

$$\text{for all } a \text{ and } b, (a/b)*b + a\%b = a.$$

but this still permits two conventions. In C++, $-13/4$ is -3 and $-13\%4$ is -1 .

4.3 Precedence and nesting parentheses

The use of parentheses (brackets) is advisable to ensure the correct evaluation of complex expressions. Here are some examples:

4 + 2 * 3 equals 10
(4+2) * 3 equals 18
-3 * 4 equals -12
4 * -3 equals -12 (but it's safer to use parentheses: 4 * (-3))
0.5(a+b) illegal (missing multiplication operator)
(a+b) / 2 equals the average value of a and b if they are of type float

The order of execution of mathematical operations is governed by rules of precedence. These are similar to those of algebraic expressions. Parentheses are always evaluated first, followed by multiplication, division and modulus operations. Addition and subtraction are last. The best thing, however, is to use parentheses (brackets) instead of trying to remember the rules.

4.4 Initialisation of variables

Variables can be assigned values when they are first defined (called initialisation):

```
type    variable = literal constant;  
float   ratio = 0.8660254;  
int     myAge = 19;  
char    answer = 'y';  
bool    raining = false;
```

The terms on the right hand side are called constants. Note the ASCII character set is represented by type char. Each character constant is specified by enclosing it between single quotes (to distinguish it from a variable name). Each char variable can only be assigned a single character. These are stored as numeric codes. (The initialisation of words and character strings will be discussed later in the section on advanced topics.)

You may assign the value of an expression too –

```
type    variable = expression;  
float   product = factor1*factor2;
```

– as long as the variables in the expression on the right hand side have already been declared and had values assigned to them. When declaring and initialising variables in the middle of a program, the variable exists (i.e., memory is assigned to store values of the variable) up to the first right brace that is encountered, excluding any intermediate nested braces, {}.

4.5 Expressions with mixed variable types

At a low level, a computer is not able to perform an arithmetic operation on two different data types of data. In general, only variables and constants of the *same* type should be combined in an expression. The compiler has strict *type checking* rules to check for this.

In cases where mixed numeric types appear in an expression, the compiler replaces all variables with copies of the highest precision type. It *promotes* them so that in an expression with integers and float variables, the integer is automatically converted to the equivalent floating

point number for the purpose of the calculation only. The value of the integer is not changed in memory. Hence, the following is legal:

```
int i=13;
float x=1.5;
x = (x * i) + 23;
```

Here the values of *i* and 23 are automatically converted to floating point numbers and the result is assigned to the float variable *x*. However the expression *i/j* here:

```
int i=13,j=4;
float result;
result = i/j;
```

is evaluated by integer division and therefore produces the incorrect assignment of 3.0 for the value of *result*. You should try to avoid expressions of this type but occasionally you will need to compute a fraction from integer numbers. In these cases the compiler needs to be told specifically to convert all the variables on the right-hand side of the assignment operator to type float or double. This is done by **casting**.

In the C++ language this is done by using the construction:

```
static_cast<type> expression
```

For example:

```
int count=13, N=400;
float fraction;
fraction = static_cast<float>(count)/N;
```

converts (casts) the value stored in the integer variable *count* into a floating point number, 13.0. The integer *N* is then promoted into a floating point number to give a floating point result.

4.6 Declaration and initialisation of symbolic constants

Like variables, **symbolic** constants have types and names. A constant is declared and initialised in a similar way to variables **but** with a specific instruction to the compiler that the value cannot be changed by the program. The values of constants must always be assigned when they are created.

```
const   type      constant-name = literal constant;
const   float     Pi = 3.14159265;
const   int       MAX = 10000;
```

The use of constants helps programmers avoid inadvertent alterations of information that should never be changed. The use of appropriate constant names instead of using the numbers also helps to make programs more readable.

5 Simple input and output

C++ does not, as part of the language, define how data is written to a screen, nor how data is read into a program. This is usually done by “special variables” (*objects*) called **input and output streams**, `cin` and `cout`, and the **insertion** and **extraction operators**. These are defined in the **header file** called `iostream`. To be able to use these *objects* and operators you must include the file `iostream` at the top of your program by including the following line of code before the main body in your program.

```
#include <iostream>
```

followed by

```
using namespace std;
```

5.1 Printing to the screen using output stream

A statement to print the value of a variable or a **string of characters** (set of characters enclosed by double quotes) to the screen begins with `cout`, followed by the **insertion operator**, (`<<`), which is created by typing the “less than” character (`<`) twice. The data to be printed follows the insertion operator.

```
cout << "text to be printed ";  
cout << variable;  
cout << endl;
```

The symbol `endl` is called a **stream manipulator** and moves the cursor to a new line. It is an abbreviation for *end of line*.

Strings of characters and the values of variables can be printed on the same line by the repeated use of the insertion operator. For example

```
int total = 12;  
cout << "The sum is " << total << endl;
```

prints out

```
The sum is 12
```

and then moves the cursor to a new line.

5.2 Input of data from the keyboard using input stream

The input stream *object* `cin` and the **extraction operator**, (`>>`), are used for reading data from the keyboard and assigning it to variables.

```
cin >> variable ;  
cin >> variable1 >> variable2 ;
```

Data typed at the keyboard followed by the *return* or *enter* key is assigned to the variable. The value of more than one variable can be entered by typing their values on the same line, separated by spaces and followed by a return, or on separate lines.

6 Control Statements

In most of the programs presented above, the statements have been sequential, executed in the order they appear in the main program.

In many programs the values of variables need to be tested, and depending on the result, different statements need to be executed. This facility can be used to **select** among alternative courses of action. It can also be used to build **loops** for the **repetition** of basic actions.

6.1 Boolean expressions and relational operators

In C++ the testing of *conditions* is done using **boolean expressions**, which yield `bool` values that are either true or false. The simplest and most common way to construct such an expression is to use the so-called **relational operators**.

<code>x == y</code>	true if x is equal to y
<code>x != y</code>	true if x is not equal to y
<code>x > y</code>	true if x is greater than y
<code>x >= y</code>	true if x is greater than or equal to y
<code>x < y</code>	true if x is less than y
<code>x <= y</code>	true if x is less than or equal to y

Be careful to avoid mixed-type comparisons. If `x` is a floating point number and `y` is an integer the equality tests will **not** work as expected.

6.2 Compound boolean expressions using logical operators

If you need to test more than one relational expression at a time, it is possible to combine the relational expressions using the **logical operators**.

Operator	C++ symbol	Example
AND	<code>&&</code> , and	<code>expression1 && expression2</code>
OR	<code> </code> , or	<code>expression1 expression2</code>
NOT	<code>!</code>	<code>!expression</code>

The meaning of these will be illustrated in examples below.

6.3 The IF selection control statement

The simplest and most common selection structure is the `if` statement, which is written in a statement of the form:

```
if(boolean-expression) statement ;
```

The `if` statement tests for a particular condition (expressed as a boolean expression) and only executes the following statement(s) if the condition is true. An example follows of a fragment of a program that tests if the denominator is not zero before attempting to calculate fraction.

```
if(total != 0)
    fraction = counter/total;
```


If the value of `total` is 0, the boolean expression above is false and the statement assigning the value of `fraction` is ignored.

If a sequence of statements is to be executed, this can be done by making a **compound statement** or **block** by enclosing the group of statements in braces.

```
if(boolean-expression)
{
    statements;
}
```

An example of this is:

```
if(total != 0)
{
    fraction = counter/total;
    cout << "Proportion = " << fraction << endl;
}
```

6.4 The IF/ELSE selection control statement

Often it is desirable for a program to take one branch if the condition is true and another if it is false. This can be done by using an **if/else** selection statement:

```
if(boolean-expression)
    statement-1;
else
    statement-2;
```

Again, if a sequence of statements is to be executed, this is done by making a compound statement by using braces to enclose the sequence:

```
if(boolean-expression)
{
    statements;
}
else
{
    statements;
}
```

An example occurs in the following fragment of a program to calculate the roots of a quadratic equation.

```

// testing for real solutions to a quadratic
d = b*b - 4*a*c;
if(d >= 0.0)
{
    // real solutions
    root1 = (-b + sqrt(d)) / (2.0*a);
    root2 = (-b - sqrt(d)) / (2.0*a);
    real_roots = true;
}
else
{
    // complex solutions
    real = -b / (2.0*a);
    imaginary = sqrt(-d) / (2.0*a);
    real_roots = false;
}

```

If the boolean condition is true, i.e. ($d \geq 0$), the program calculates the roots of the quadratic as two real numbers. If the boolean condition is false, then a different sequence of statements is executed to calculate the real and imaginary parts of the complex roots. Note that the variable `real_roots` is of type `bool`. It is assigned the value `true` in one of the branches and `false` in the other.

6.5 ELSE IF multiple selection statement

Occasionally a decision has to be made on the value of a variable that has more than two possibilities. This can be done by placing `if` statements within other `if-else` constructions. This is commonly known as *nesting* and a different style of indentation is used to make the multiple-selection functionality much clearer. This is given below:

```

if(boolean-expression-1)
    statement-1;
else if(boolean-expression-2)
    statement-2;
:
else
    statement-N;

```

For compound statement blocks, braces must be used.

6.6 SWITCH multiple selection statement

Instead of using multiple if/else statements C++ also provides a special control structure, switch.

For a variable *x* the `switch(x)` statement tests whether *x* is equal to the constant values *x1*, *x2*, *x3*, etc. and takes appropriate action. The `default` option is the action to be taken if the variable does not have any of the values listed.

```
switch(x)
{
    case x1:
        statements1;
        break;

    case x2:
        statements2;
        break;

    case x3:
        statements3;
        break;

    default:
        statements4;
        break;
}
```

The `break` statement causes the program to proceed to the first statement after the switch structure. Note that the switch control structure is different to the others in that braces are not required around multiple statements.

The following example uses the switch statement to produce a simple calculator that branches depending on the value of the operator being typed in. The operator is read and stored as a character value (`char`). The values of `char` variables are specified by enclosing them between single quotes. The program is terminated (`return -1`) if two numbers are not input or the simple arithmetic operator is not legal. The return value of `-1` instead of `0` signals that an error took place.

```

// CalculatorSwitch.cc
// Simple arithmetic calculator using switch() selection.

#include <iostream>
using namespace std;

int main()
{
    float a, b, result;
    char operation;

    // Get numbers and mathematical operator from user input
    cin >> a >> operation >> b;

    // Character constants are enclosed in single quotes
    switch(operation)
    {
        case '+':
            result = a + b;
            break;

        case '-':
            result = a - b;
            break;

        case '*':
            result = a * b;
            break;

        case '/':
            result = a / b;
            break;

        default:
            cout << "Invalid operation. Program terminated." << endl;
            return -1;
    }

    // Output result
    cout << result << endl;
    return 0;
}

```

6.7 The WHILE repetition control statement

Repetition control statements allow the programmer to specify actions that are to be repeated while some condition is true. In the while repetition control structure:

```
while(boolean-expression)
{
    statements;
}
```

the boolean expression (or 'condition') is tested and the statements enclosed by the braces are executed repeatedly while the condition given by the boolean expression is true. The loop terminates as soon as the boolean expression is evaluated and tests false. Execution will then continue on the first line after the closing brace. You can have as many statements as you like inside the loop, including one or zero.

Note that if the boolean expression is initially false the statements are not executed. In the following example the boolean condition becomes false when the first negative number is input at the keyboard. The sum is then printed.

```
// AddWhilePositive.cc
// Computes the sum of numbers input at the keyboard.
// The input is terminated when input number is negative.

#include <iostream>
using namespace std;

int main()
{
    float number, total=0.0;

    cout << "Input numbers to be added: " << endl;
    cin >> number;

    // Stay in loop while input number is positive
    while(number >= 0.0)
    {
        total = total + number;
        cin >> number;
    }

    // Output sum of numbers
    cout << total << endl;
    return 0;
}
```

6.8 Increment and decrement operators

Increasing and decreasing the value of an integer variable is a commonly used method for counting the number of times a loop is executed. C++ provides a special operator `++` to increase the value of a variable by 1. The following are equivalent ways of *incrementing* a counter variable by 1.

```
count = count + 1;
count++;
```

The operator `--` decreases the value of a variable by 1. The following are both *decrementing* the counter variable by 1.

```
count = count - 1;
count--;
```

C++ also provides the operators `+=` and `-=`, which increase and decrease the value of a variable by an expression. For example:

```
count -= 5;
total += x + y;
```

the first line decreases `count` by 5; the second increases `total` by the value of the expression `x+y`.

6.9 The FOR repetition control statement

Often in programs we know how many times we will need to repeat a loop. A `while` loop could be used for this purpose by setting up a starting condition; checking that a condition is true; and then incrementing or decrementing a counter within the body of the loop. For example we can adapt the `while` loop in `AddWhilePositive.cc` so that it executes the loop `N` times and hence sums the `N` numbers typed in at the keyboard.

```
i=0;           // initialise counter
while(i<N)     // test whether counter is still less than N
{
    cin >> number;
    total = total + number;
    i++;       // increment counter
}
```

The initialisation, test and increment operations of the `while` loop are so common when executing loops a fixed number of times that C++ provides a concise representation – the `for` repetition control statement:

```
for(int i=0; i<N; i++)
{
    cin >> number;
    total = total + number;
}
```

This control structure has exactly the same effect as the `while` loop listed above.

The following is a simple example of a for loop with an increment statement using the increment operator to calculate the sample mean and variance of N numbers. The for loop is executed N times.

```
//ForStatistics.cc
//Computes the sample mean and variance of N numbers input at the keyboard.
//N is specified by the user but must be 10 or fewer in this example.

#include <iostream>
using namespace std;

int main()
{
    int    N=0;
    float  number, sum=0.0, sumSquares=0.0;
    float  mean, variance;

    // Wait until the user inputs a number in correct range (1-10)
    // Stay in loop if input is outside range
    while(N<1 || N>10)
    {
        cout << "Number of entries (1-10) = ";
        cin  >>  N;
    }

    // Execute loop N times. Start with i=1; increment i at end of each loop;
    //                                     exit loop when i = N+1.
    for(int i=1; i<=N; i++ )
    {
        cout << "Input item number " << i << " = ";
        cin >> number;
        sum = sum + number;
        sumSquares = sumSquares + number*number;
    }

    mean = sum/N;
    variance = sumSquares/N - mean*mean;

    cout << "The mean of the data is " << mean <<
        " and the variance is " << variance << endl;
    return 0;
}
```

A more general form of the for repetition control statement is given by:

```
for(statement1; boolean-expression; statement3)
{
    statement2;
}
```

The first statement (*statement1*) is executed and then the boolean expression (condition) is tested. If it is true *statement2* (which may be more than one statement) and *statement3* are executed. The boolean expression (condition) is then tested again and if true, *statement2* and *statement3* are repeatedly executed until the condition given by the boolean expression becomes false.

On page 38, we gave an example where we executed a loop N times using

```
for(int i=0; i<N; i++) {    ...    }
```

In the `ForStatistics.cc` example, we used this form to loop N times.

```
for(int i=1; i<=N; i++) {    ...    }
```

Notice the two differences between these two commands.

The first example starts out with $i=0$ and continues looping as long as $i<N$, which means that the values that i takes on in the loop are $0, 1, \dots, N-1$. This is a very common style in C++ – to start at zero, and go up to $N - 1$.

The second example starts out with $i=1$ and continues looping as long as $i\leq N$, which means that the values that i takes on in the loop are $1, 2, \dots, N$. After we exit the loop, i will be equal to $N+1$.

Beware! One of the most common programming errors is to mix up the ‘zero-offset’ convention and the ‘one-offset’ convention.

6.10 The DO...WHILE repetition control statement

Both the `while` and the `for` repetition methods check the condition *before* every iteration. This means that it’s possible that the body of the loop may never be executed – if the condition is already not true at the start. Sometimes this is exactly what you want. For example, if you want to find how many times a positive integer n is exactly divisible by 2, you could use:

```
i=0;    while((n%2)==0) { n /= 2; i ++ ; }
```

or

```
i=0;    for(;;(n%2)==0;) { n /= 2; i ++ ; }
```

On the other hand, sometimes you want the body of the loop always to be executed at least once, and you want the condition to be tested only before subsequent iterations. In such cases, you can use the construction

```
do
{
    body statements ;
} while ( boolean-expression ) ;
```

Notice the final semicolon after the condition, in contrast to `for` and `while`.

A good example of this is a ‘please enter a number’ dialogue, where the question must be asked once, and perhaps repeated *until* the user supplies a valid answer.

```
do {
    cout << "Enter a number between 1 and 10:" ;
    cin >> N ;
} while( (N<1) || (N>10) ) ;
```

In English, this says ‘ask for a number N , repeatedly, as long as the number N is *outside* the range’. Remember, the two-pipes symbol ‘`||`’ is the logical operator OR.

A. Objectives

After reading through sections 3–5 of the tutorial guide and working through the examples you should be able to:

- Declare and define variables and constants
- Assign values to variables and manipulate them in arithmetic expressions
- Write the value of variables to the screen
- Read in the value of variables from the keyboard
- Write simple programs that use loops.

You will now use these skills to write and execute the program described in the following computing exercise (section B). Details and hints on the writing and *debugging* of the programs are given in section C.

B. Computing Exercises

Modify the `SimpleAdder.cc` program so that it converts a physical quantity in one set of units to another set of units and displays the result as a floating point number. For example, read in a time in years, and display that time in seconds; or read in a power in kWh per day and display it in W.

Write a program that uses a `for` loop to sum the first N odd numbers, where N is a number supplied by the user. When it has computed the sum, your program should check that

$$\sum_{i=1}^N (2i - 1) = N^2$$

Write a program that uses a loop to print out the sequence of floating-point numbers:

$$1, 1 + \frac{1}{1}, 1 + \frac{1}{1 + \frac{1}{1}}, 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}, 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}}, \dots$$

(Print out the first 20 terms.) If you would like to see more digits of precision than the default (6), see the output formatting instructions on page 83.

Modify your program so that it reads in a number x from the user, then prints out the sequence of floating-point numbers:

$$x, x + \frac{1}{x}, x + \frac{1}{x + \frac{1}{x}}, x + \frac{1}{x + \frac{1}{x + \frac{1}{x}}}, x + \frac{1}{x + \frac{1}{x + \frac{1}{x + \frac{1}{x}}}}, \dots$$

What does this sequence

$$2, 2 + \frac{1}{2}, 2 + \frac{1}{2 + \frac{1}{2}}, 2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}, 2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}}, \dots$$

tend to?

Write a program that uses a loop to print out the sequence of floating-point numbers:

$$\sqrt{1}, \sqrt{1 + \sqrt{1}}, \sqrt{1 + \sqrt{1 + \sqrt{1}}}, \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1}}}}, \dots$$

What does this sequence tend to? The square root function in C++ is called `sqrt(...)`. To tell the C++ compiler that you want to use maths functions such as `sqrt` you must put `#include <cmath>` at the top of your program, before or after the `#include <iostream>` statement.

Optional: Write a program that prints out all prime numbers less than or equal to N . Hint: use the modulo division operator (%).

C. Tips

1. Start with a working program

The easiest way to write programs is to modify the existing programs. Make small modifications, compiling whenever possible.

2. Making modifications

- Mentally check that the program is doing what it is required to do.
- Check that your program is laid out correctly with *indentation* at the beginning and *semi-colons* at the end of each statement.
- Include comment statements to help you and others understand what the program is doing.

3. Checking syntax

Compile, using `g++ -Wall` or `make`. If there are no syntax errors, the program will compile successfully. One or more errors will cause the compiler to print out error or warning messages, which will be listed along with program line numbers indicating where the error occurred. **Remember the tips on page 24** – always use *warnings* when compiling. And save typing by using `make`.

4. Compilation errors

Don't be worried by the number of error messages. The compiler is trying to give as much help as it can. You can get a lot of error messages from just one mistake so **always look at the first error message first**.

Find the line in the file to which it refers (there is a Goto-line option on the editor Edit menu) and try to correct it. Often, the compiler will report that the error is in a line just *after* the one in which there was a mistake. If you cannot spot the mistake straight away, look at neighbouring lines.

The most common errors will be due to undeclared variables or variables incorrectly declared, and missing or incorrect punctuation. If you are feeling stuck, a really good way to track down the cause of an error message is to explain to a friend, step by step, how you modified a compilable program to get the program with the compilation error.

5. Debugging tips

You don't need to close the editor each time you save the file and recompile; you should have only one copy of Emacs running at any time.

Continue with this compile/debug/edit procedure until your program compiles successfully.

6. Finding logical errors

Run your program and check that it gives the correct answer.

As in session 1, here's the self-assessed bit.

Set yourself an **additional programming task** along the lines of the preceding tasks, and solve it by yourself. Then submit your task statement and your solution to the course pigeonhole.

Put your task statement and solution file in a new folder (IB.2) inside your existing folder – like this (if your CRSID is jb007)

```
mkdir /ux/physics/part_1b/c++/pigeonhole/jb007/IB.2
cp README /ux/physics/part_1b/c++/pigeonhole/jb007/IB.2
cp blah.cc /ux/physics/part_1b/c++/pigeonhole/jb007/IB.2
```

Please do not put enormous files in the course pigeonhole – just small text files and .cc files.

Deadline: You are strongly encouraged to complete this work on time during week 6. See page 5 for the final deadlines.

If you finish session 2 with time to spare, make a start on session 3.

A. Objectives

After reading through section 6 of the tutorial guide and studying and executing the C++ programs in the examples (boxed) you should now be familiar with:

- Boolean expressions with relational operators
- Simple control structures for selection and repetition

You will now use these skills in the following computing exercises. You'll also learn to use the graphing program, gnuplot.

B. Computing Exercises

Reminder: programming in pairs is a very good idea, *as long as the weaker programmer does the typing.*

Simple Monte Carlo

Monte Carlo methods use random numbers to generate approximate answers to mathematical questions. A good Monte Carlo method has the property that the more random numbers are used, the more accurate the answer becomes.

The program `RandDemo6.cc` on the next page shows how to generate N pairs (x, y) uniformly distributed in the unit square, and test whether each pair is in the unit circle or not. The variable N is supplied by the user on the command line. (I recommend this non-interactive method of getting inputs from the user, rather than `cin`.) This program has several other new features, some of which we'll explain later. For the moment, don't worry about the lines with `#include`, `#define`, `using`, or `scanf`.

Make sure you understand everything inside the main for loop.

The program uses the 'ternary conditional operator' `"?"` in the assignment

```
outcome = ( x*x + y*y < 1.0 ) ? 1 : 0
```

In general,

```
outcome = ( expression1 ) ? expression2 : expression3
```

means that `outcome` is set equal to `expression2` if `expression1` is true, and to `expression3` if `expression1` is false.

Rewrite the program so as to use an if-else in place of this ternary conditional. Explain to a friend how the program works.

When `RandDemo6` runs, the last two columns of its output contain the values of N and `4.0 * fraction_in`.

What do you expect the value of `4.0*fraction_in` to do as N gets big?

```

// RandDemo6.cc
//  usage:
//    ./RandDemo6 [<N> [<seed>]]
//  example:
//    ./RandDemo6                      (Run with default value of N and seed)
//    ./RandDemo6 30 9237832          (Run with N=30 and seed=9237832)
//  * uses random() to get a random integer
//  * gets parameters from command line

#include <iostream>
using namespace std;

#define ranf() \
    ((double)random()/(1.0+(double)0x7fffffff)) // Uniform from interval [0,1)
                                                // obtained by division by 2**31

int main(int argc, char* argv[])
{
    int    outcome, N=10, count_in=0 , seed=123 ;
    double fraction_in ;

    if(argc>1)  {
        sscanf( argv[1], "%d", &N ) ; // put the first command-line argument in N
    }
    if(argc>2) {
        sscanf( argv[2], "%d", &seed ) ; // put the 2nd argument in seed
    }
    // Write out a summary of parameters
    cout << "# " << argv[0] << " N=" << N << " seed=" << seed << endl ;

    // Initialise random number generator
    srandom(seed);

    // Perform N experiments.
    for(int n=1; n<=N; n++) {
        double x = ranf();      // ranf() returns a real number in [0,1)
        double y = ranf();
        outcome = ( x*x + y*y < 1.0 ) ? 1 : 0 ;
        count_in += outcome ;
        //Integer variables must be converted (cast) for correct division
        fraction_in = static_cast<double>(count_in)/n;
        cout << "Location" << outcome << "\t" << x << "\t" << y << "\t"
        << count_in << "\t" << n << "\t" << 4.0 * fraction_in << endl;
    }
    return 0;
}

```

When you are running a program in a terminal (also known as a shell), the symbol > can

be used to redirect the output of the program to a file. Send the output of RandDemo6 to a file, using, say,

```
./RandDemo6 20 2389 > tmpfile
```

Start gnuplot by typing gnuplot in a terminal. When it has started, gnuplot gives you a prompt, which looks like this: gnuplot>. Plot the generated points (x, y) using, for example,

```
gnuplot> plot 'tmpfile' u 2:3 w points
```

What we mean here is “please type plot 'tmpfile' u 2:3 w points”. gnuplot has many options for making nice plots. Type help to get help. You can change the options interactively – for example,

```
gnuplot> set size square
gnuplot> replot
```

– but I recommend writing the gnuplot commands in a plain text file – say splat.gnu – and then loading the commands thus

```
gnuplot> load 'splat.gnu'
```

Putting the commands in a file has the advantage that you then have a permanent record of how a plot was generated, should you want to replot or modify it in the future.

Unix provides many utilities for dealing with textfiles. When you are doing scientific computation, we recommend that you write the results of the computation out in plain text files, then handle subsequent inspection or processing of the results with other programs – rather than writing a single monolithic program that does everything. Here’s an example. First run in your terminal:

```
./RandDemo6 200 1234321 > tmpfile
```

– which puts 200 outcomes in tmpfile. Now in your terminal run

```
grep Location1 tmpfile > file1
grep -v Location1 tmpfile > file0
```

The first grep command looks at every line in tmpfile and prints out the lines that contain the string Location1. The output went into file1 because of the redirect (>). You can check what happened with the command:

```
more file1
```

The second command uses grep with a special flag -v, which means ‘invert the match’. Every line in tmpfile that does *not* contain the string Location1 gets printed.

Use file0, file1, and gnuplot to make a pretty picture that you could use to explain to someone how this Monte Carlo method can be used to estimate π . By the way, gnuplot can easily plot functions as well as datafiles. For example,

```
gnuplot> plot sqrt(1-x*x) with line 3
```

plots the function $\sqrt{1 - x^2}$ with a line of colour '3' (which might be blue). gnuplot always calls the variable on the horizontal axis x. You can grab a screenshot of your pretty picture with the terminal command: `import pretty.png`.

Having generated N points, what's the estimate of π ? How close do you expect this estimate to be to the true value? **Write down the expected error $\sigma(N)$ of the estimate as a function of N .**

Use gnuplot to plot $\pi \pm \sigma(N)$ as a function of N .

You can define and plot functions in gnuplot like this:

```
gnuplot> f(N) = 1.0/N
gnuplot> set xrange [1:200]
gnuplot> plot pi+f(x) with line 5, pi-f(x) with line 5
```

NB: the function $f(N)$ is *not* the answer to the question above.

Now superpose your plot of the theoretical prediction about the estimate $[\pi \pm \sigma(N)]$ on the Monte Carlo estimate (column 6) as a function of N . The estimate can be plotted with

```
plot 'tmpfile' u 5:6 with linespoints
```

Does the Monte Carlo method work as expected? How many points are required to get an answer that's probably accurate to 1%? Do a couple of runs of that length, putting the outputs in separate files (say tmpfile1 and tmpfile2), and see what happens. Zoom in on the interesting yrange. (You may be able to zoom in to a plot by right-clicking on the gnuplot display. Alternatively, use `set yrange [3.1:3.2]`.)

The output files may get rather large, and slow to plot. **Modify your C++ program so that rather than printing a line for every outcome, it prints a gradually decreasing fraction of the lines as n increases.**

How many points are required to get an answer that's probably accurate to 2 decimal places? To 0.1%? Make a prediction, then check it.

Now, here's the exercise whose solution you submit to the course pigeonhole.

Write a program that estimates $\ln(2)$ to 2 decimal places by a Monte Carlo method.

Hint: you may find the function $y = 1/x$ interesting to think about.

If you solve this problem by yourself, please submit your solution as follows: write a file README that briefly describes your approach, and a statement that you made the solution yourself; put that file and your .cc file in a new folder (IB.3) inside your pigeonhole folder – like this (if your CRSID is jb007)

```
mkdir /ux/physics/part_1b/c++/pigeonhole/jb007/IB.3
cp README /ux/physics/part_1b/c++/pigeonhole/jb007/IB.3
cp blah.cc /ux/physics/part_1b/c++/pigeonhole/jb007/IB.3
```

If on the other hand, you didn't solve the $\ln(2)$ exercise by yourself, please set yourself an **additional programming task** along the lines of the preceding tasks, and solve it by yourself. Then submit your task statement and your solution to the course pigeonhole.

Deadline: You are strongly encouraged to complete this work on time during week 7. See page 5 for the final deadlines.

More efficient Monte Carlo (optional exercise)

How can you modify the Monte Carlo method to reduce the number of random numbers required to obtain $\ln(2)$ to a given accuracy?

Comment

A Monte Carlo approach seems a pretty goofy way to evaluate an integral, but in high-dimensional problems in statistical physics, quantum physics, and statistical inference, Monte Carlo methods are very widely used. See *Information Theory, Inference, and Learning Algorithms*, David J.C. MacKay, Cambridge University Press (2003) – available free online at www.inference.phy.cam.ac.uk/mackay/itila or tinyurl.com/d6vck – for further reading about modern Monte Carlo methods.

Testing for randomness (optional exercise)

Returning to the points (x, y) that you plotted a couple of pages ago: do you think these plotted points (x, y) *look* like perfectly random points? If you think *not*, define a measure of non-randomness that captures what you think is wrong with the so-called random points, and write a program to measure this quantity for the points generated by the `random()` routine.

Estimating the value of π II (optional exercise)

Write a program to print out the first N terms in the series:

$$\sum_{i=1}^N \frac{1}{i^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots + \frac{1}{N^2}$$

The number of terms, N , is to be input by the user at the keyboard or on the command line.

Modify your program so that for each value of the index i , it evaluates the sum of the first i terms.

The sum of this series can be shown (Leonhard Euler (1707-1783)) to converge to $\pi^2/6$. Make another modification to your program so that at each *iteration* the estimate of π is printed alongside the sum. [To use the mathematical library function `sqrt()` you must include the header file `cmath` at the top of your program.] Plot the estimate as a function of N . How good is the estimate of π after $N=100$ iterations? How many iterations are needed to get an estimate of π that is accurate to 2 decimal places?

7 Functions

Computer programs that solve real-world problems are usually much larger than the simple programs discussed so far. To design, implement and maintain larger programs it is necessary to break them down into smaller, more manageable pieces or *modules*. Dividing the problem into parts and building the solution from simpler parts is a key concept in problem solving and programming.

In C++ we can subdivide the functional features of a program into blocks of code known as **functions**. In effect these are subprograms that can be used to avoid the repetition of similar code and allow complicated tasks to be broken down into parts, making the program modular.

Until now you have encountered programs where all the code (statements) has been written inside a single function called `main()`. Every executable C++ program has at least this function. In the next sections we will learn how to write additional functions.

7.1 Function definition

Each function has its own name, and when that name is encountered in a program (the **function call**), execution of the program branches to the body of that function. After the last statement of the function has been processed (the **return** statement), execution resumes on the next line after the call to the function.

Functions consist of a header and a body. The **header** includes the name of the function and tells us (and the compiler) what type of data it expects to receive (the *parameters*) and the type of data it will return (*return value type*) to the calling function or program.

The **body** of the function contains the instructions to be executed. If the function returns a value, it will end with a return statement. The following is a formal description of the syntax for defining a function:

```
return-value-type function-name (parameter-list)
{
    declaration of local variables;
    statements;

    return return-value;
}
```

The syntax is very similar to that of the main program, which is also a function. `main()` has no parameters and returns the integer 0 if the program executes correctly. Hence the return value type of the `main()` function is `int`.

```
int main()
{
    declarations;
    statements;

    return 0;
}
```

7.2 Example of function definition, declaration and call

Let us first look at an example of program written entirely in the function `main()` and then we will modify it to use an additional **function call**.

We illustrate this with a program to calculate the factorial ($n!$) of an integer number (n) using a for loop to compute:

$$n! = 1 \cdot 2 \cdot 3 \dots (n-2) \cdot (n-1) \cdot n$$

```
// MainFactorial.cc
// Program to calculate factorial of a number

#include <iostream>
using namespace std;

int main()
{
    int number=0, factorial=1;

    // User input must be an integer number between 1 and 10
    while(number<1 || number>10)
    {
        cout << "Enter integer number (1-10) = ";
        cin >> number;
    }

    // Calculate the factorial with a FOR loop
    for(int i=1; i<=number; i++)
    {
        factorial = factorial*i;
    }

    // Output result
    cout << "Factorial = " << factorial << endl;
    return 0;
}
```

Even though the program is very short, the code to calculate the factorial is best placed inside a function since it is likely to be executed many times in the same program or in different programs (e.g., calculating the factorials of many different numbers, computing binomial coefficients and permutations and combinations).

```

// FunctionFactorial.cc
// Program to calculate factorial of a number with function call

#include <iostream>
using namespace std;

// Function declaration (prototype)
int Factorial(int M);

int main()
{
    int number=0, result;

    // User input must be an integer number between 1 and 10
    while(number<1 || number>10)
    {
        cout << "Integer number = ";
        cin >> number;
    }

    // Function call and assignment of return value to result
    result = Factorial(number);

    //Output result
    cout << "Factorial = " << result << endl;
    return 0;
}

// Function definition (header and body)
// An integer, M, is passed from caller function.
int Factorial(int M)
{
    int factorial=1;

    // Calculate the factorial with a FOR loop
    for(int i=1; i<=M; i++)
    {
        factorial = factorial*i;
    }

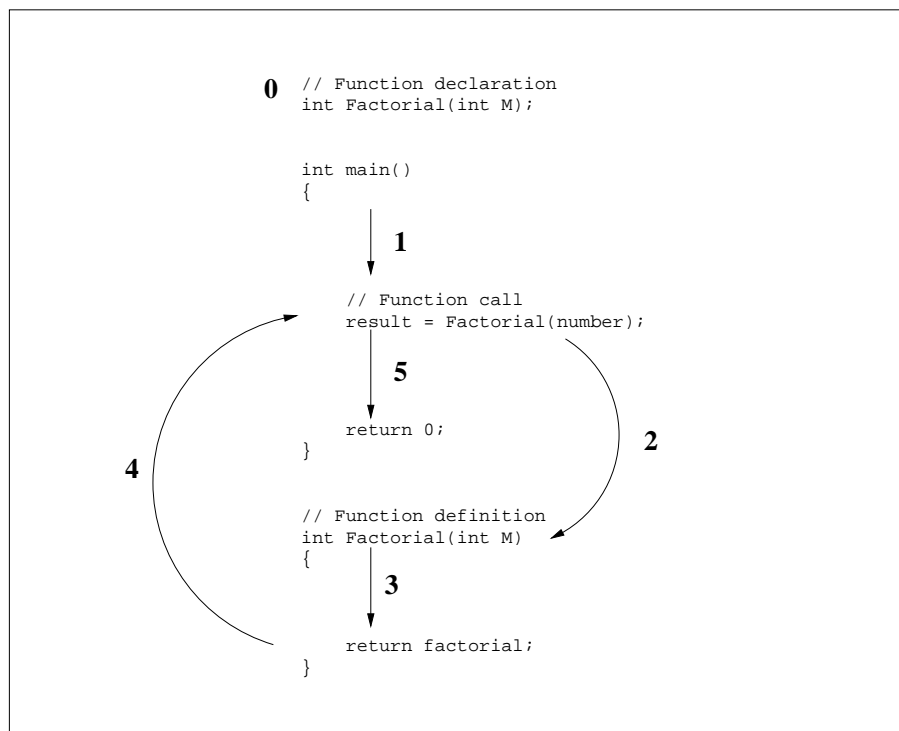
    return factorial; // This value is returned to caller
}

```

Three modifications to the program have been made to incorporate a function. If we look at the modified sample program, FunctionFactorial.cc, we find:

1. The **declaration** of the function above the main program. The declaration (also known as the *prototype*) tells the compiler about the function and the *type* of data it requires and will *return* on completion.
2. The **function call** in the main body of the program determines when to branch to the function and how to return the value of the data computed back to the main program.
3. The **definition** of the function `Factorial()` below the main program. The definition consists of a **header**, which specifies how the function will interface with the main program, and a **body**, which lists the statements to be executed when the function is called.

Before a function can be used it must be declared (0), usually at the top of the program file.¹ When the function name is encountered in the main body of the program (1), execution branches to the body of the function definition (2). Copies of the values of function arguments are stored in the memory locations allocated to the parameters. The statements of the function are then executed (3) up to the first return statement when control returns to the main body (4). Any value returned by the function is stored by an assignment statement. Execution in the main body is resumed immediately after the function call (5).



7.3 Function header and body

The function is defined below the body of `main()`. The **header** in this example:

```
int Factorial(int M)
```

¹Strictly speaking, it's not always essential to **declare** a function before it is used, as long as the compiler encounters the function's **definition** before the function is used.

indicates that the `Factorial()` function expects to be **passed** an integer value (the *parameter type*) from the main body of the program and that the value passed will be stored locally in a variable named `M` (the *formal parameter name*). The *return value type* of the function is also `int` in this example, indicating that at the end of executing the body of the function, an integer value will be returned to the statement in which the function was called. Functions that do not return a value have *return value type* `void`.

The **body** of the function computes the factorial of a number in exactly the same way as in the example with only a `main()` function. The execution of the function terminates with a *return statement*:

```
return factorial;
```

which specifies that the value stored in the function variable `factorial` should be passed back to the calling function.

7.4 Function declaration

Every function should be **declared** before it is used. The declaration tells the compiler the name, return value type and parameter types of the function. In this example the declaration:

```
int Factorial(int M);
```

tells the compiler that the program passes the value of an integer to the function and that the *return value* must be assigned to an integer variable. The declaration of a function is called its **prototype**, which means the “first” time the function is identified to your program.

The function prototype and the function definition must agree exactly about the return value type, function name and the parameter types. The function prototype is usually a copy of the function header followed by a semicolon to make it a declaration and placed before the main program in the program file.

7.5 Function call and execution

The function definition is entirely passive. By itself it does nothing unless instructed to execute. This is done by a statement in the main program called the **function call**.

For example the statement:

```
result = Factorial(number);
```

calls the function `Factorial()` and passes a copy of the value stored in the variable, `number`. When the function is called, computer memory is allocated for the parameter, `M`, and the value passed is copied to this memory location. Memory is also allocated to the (local) variables `factorial` and `i`. The statements of the function are then executed and assign a value to the variable `factorial`. The return statement passes this value back to the calling function. The memory allocated to the parameters and local variables is then destroyed. The value returned is assigned to the variable on the left-hand side, `result`, in the expression used to call the function. The net effect of executing the function in our example is that the variable `result` has been assigned the value of the factorial of `number`.

A function can be called any number of times from the same or different parts of the program. It can be called with different parameter values (though they must be of the correct type). For example the following fragment of code can be used to print out the factorials of the first 10 integers:

```

for(int i=1; i<=10; i++)
{
    result = Factorial(i);
    cout << i << "! = " << result << endl;
}

```

and:

```

binomialCoefficient = Factorial(n)/(Factorial(k) * Factorial(n-k));

```

can be used to compute the binomial coefficient:

$$\frac{n!}{k!(n-k)!}$$

(though this is *not* the best way to compute a binomial coefficient!).

7.6 Function arguments

The names of variables in the statement calling the function will not in general be the same as the names in the function definition, although they must be of the same type. We often distinguish between the **formal parameters** in the function definition (e.g. `M`) and the **actual parameters** for the values of the variables passed to the function (e.g. `number` in the example above) when it is called.

Function arguments (actual parameters) can include constants and mathematical expressions. For example the following statement assigns the value 24 to the variable `result`.

```

result = Factorial(4);

```

The function arguments can also be functions that return an appropriate value (for example `Factorial(Factorial(4))`), although this concise style makes the code harder to read and debug.

7.7 Another example

Here is another example of the declaration, definition and call of a function, `AreaTriangle()`, in a program to calculate the area of a regular hexagon inscribed in a circle of radius input by the user.

```
// HexagonValue.cc
// Program to calculate the area of a regular hexagon inscribed in a
// circle as sum of areas of 6 triangles by calling AreaTriangle()

#include <iostream>
using namespace std;

// AreaTriangle function prototype
float AreaTriangle(float base, float height);

int main()
{
    float side, segmentHeight, hexagonArea;
    float cosTheta = 0.866025;

    cout << "Program to calculate the area of a hexagon" << endl;
    cout << "Enter side of hexagon: ";
    cin >> side;

    // Base of triangle is equal to side, height is side*cos(30)
    segmentHeight = side*cosTheta;

    // Function returns area of segment. 6 segments for total area.
    hexagonArea = 6.0 * AreaTriangle(side,segmentHeight);

    cout << "Area of hexagon = " << hexagonArea << endl;
    return 0;
}

// AreaTriangle function definition
float AreaTriangle(float base, float height)
{
    float area;

    area = (base*height)/2.0;
    return area;
}
```

The statement:

```
hexagonArea = 6.0 * AreaTriangle(side,segmentHeight);
```

calls the function to calculate the area of a triangle with base and height given by the values

stored in `side` and `segmentHeight` and then assigns the value of 6 times the area (the return value of the function) to the variable `hexagonArea`. It is therefore equivalent to the following:

```
segmentArea = AreaTriangle(side,segmentHeight);  
hexagonArea = 6.0*segmentArea;
```

7.8 Passing by value or reference

There are two ways to pass values to functions. Up to now we have looked only at examples of **passing by value**. In the *passing by value* way of passing parameters, a *copy* of the variable is made and passed to the function. Changes to that copy do not affect the original variable's value in the calling function. This prevents the accidental corrupting of variables by functions and so is often the preferred method for developing correct and reliable software systems. One disadvantage of *passing by value* however, is that only a single value can be returned to the caller. If the function has to modify the value of an argument or has to return more than one value, then another method is required.

An alternative uses **passing by reference** in which the function is told where in memory the data is stored (i.e., the function is passed the memory address of the variables). In passing the address of the variable we allow the function to not only read the value stored but also to change it.

Here's an analogy. When you tell a friend about a webpage on the course wiki, you can either print out a *copy* of the webpage for them; or you can send them the URL of the webpage. Giving them the copy is 'pass by value'; giving them the URL is 'pass by reference'. When you 'pass by value', they can manipulate their copy of the page, and their manipulations have no effect on the original page. When you 'pass by reference', they will be able to edit the contents of the webpage.

There are two ways to pass a parameter by reference. The simple (and rather subtle) way is described here in the example `SortReference.cc`. An alternative approach to passing by reference is demonstrated on the web in `SortReference2.cc`. (<http://tinyurl.com/38fp68>)

To indicate that a function parameter is *passed by reference* the symbol `&` is placed next to the variable name in the parameter list of the function definition and prototype (but **not** the function call). Inside the function the variable is treated like any other variable. Any changes made to it, however, will automatically result in changes in the value of the variable in the calling function.

A simple example of passing by reference is given below for a function that swaps the values of two variables in the calling function's data by reading and writing to the memory locations of these variables. Note that the parameters are mentioned only by name in the function call. This appears to be the same as *calling by value*. The function header and prototype, however, must use the `&` symbol by the variable name to indicate that the call is by reference and that the function can change the variable values.


```

// SortReference.cc
// Program to sort two numbers using call by reference.
// Smallest number is output first.

#include <iostream>
using namespace std;

// Function prototype for call by reference
void swap(float &x, float &y);

int main()
{
    float a, b;

    cout << "Enter 2 numbers: " << endl;
    cin >> a >> b;
    if(a>b)
        swap(a,b);

    // Variable a contains value of smallest number
    cout << "Sorted numbers: ";
    cout << a << " " << b << endl;
    return 0;
}

// A function definition for call by reference
// The variables x and y will have their values changed.

void swap(float &x, float &y)
// Swaps x and y data of calling function
{
    float temp;

    temp = x;
    x = y;
    y = temp;
}

```

8 Math library and system library built-in functions

Functions come in two varieties. They can be defined by the user or built in as part of the compiler package. As we have seen, user-defined functions have to be declared at the top of the file. Built-in functions, however, are declared in **header files** using the `#include` directive at the top of the program file, e.g. for common mathematical calculations we include the file `cmath` with the `#include <cmath>` directive, which contains the *function prototypes* for the mathematical functions in the `cmath` library.

8.1 Mathematical functions

Math library functions allow the programmer to perform a number of common mathematical calculations:

Function	Description
<code>sqrt(x)</code>	square root
<code>sin(x)</code>	trigonometric sine of x (in radians)
<code>cos(x)</code>	trigonometric cosine of x (in radians)
<code>tan(x)</code>	trigonometric tangent of x (in radians)
<code>exp(x)</code>	exponential function
<code>log(x)</code>	natural logarithm of x (base e)
<code>log10(x)</code>	logarithm of x to base 10
<code>fabs(x)</code>	absolute value (unsigned)
<code>ceil(x)</code>	rounds x up to nearest integer
<code>floor(x)</code>	rounds x down to nearest integer
<code>pow(x,y)</code>	x raised to power y

8.2 Random numbers

Other **header files** that contain the function prototypes of commonly used functions include `cstdlib` and `time`. These contain functions for generating random numbers and for manipulating time and dates respectively.

The function `random()` *randomly* generates an integer between 0 and the maximum value that can be stored as an integer. Every time the function is called:

```
randomNumber = random();
```

a different number will be assigned to the variable `randomNumber`. Each number is supposed to have an equal chance of being chosen each time the function is called. The details of how the function achieves this will not be discussed here.

Before a random number generator is used for the first time it should be initialised by giving it a number called the *seed*. Each seed will result in a different sequence of numbers. The function `srandom()` initialises the random number generator, `random()`. It must be called with an arbitrary integer parameter (the *seed*). If you want fresh random numbers every time, you can conveniently generate a fresh seed by using the value returned by the system clock function `time()` with the parameter `NULL`. This returns the calendar time in seconds, converted to an integer value. The following call, which is usually used only once, can be used to initialise the random number generator:

```
srandom(time(NULL));
```

Alternatively, if you want to make random experiments *that can be exactly reproduced*, you might prefer to give the user explicit control of the seed.

The `RandDemo6.cc` program used these system functions. Here's another example.

```
// Function to simulate rolling a single 6-sided die.  
// Function takes no arguments but returns an integer.  
// Each call will randomly return a different integer between 1 and 6.  
  
int RollDie()  
{  
    int randomNumber, die;  
  
    randomNumber = random();  
    die = 1 + (randomNumber % 6);  
    return die;  
}
```

A. Objectives

After reading through sections 7 and 8 of the tutorial guide and studying and executing the C++ programs in the examples (boxed) you should now be familiar with

- Definition, declaration and calling of functions
- Passing values to and returning values from functions
- Math and system library functions

You will now use these skills in the following computing exercises. The information in sections C and D will help you to think about what is involved in producing a working solution. Suggestions for the *algorithms* (mathematical recipes) and their implementation in C++ are provided.

B. Computing Exercises

Finding a solution to $f(x) = 0$ by iteration

Write a function that computes the square root of a number in the range $1 < x \leq 100$ with an accuracy of 10^{-4} using the bisection method. The Math library function *must not* be used.

Test your function by calling it from a program that prompts the user for a single number and displays the result.

Modify this program so that it computes the square roots of numbers from 1 to 10. Compare your results with the answers given by the `sqrt()` mathematical library function.

C. The bisection method

The problem of finding the square root of a number, c , is a special case of finding the root of a non-linear equation of the form $f(x) = 0$ where $f(x) = c - x^2$. We would like to find values of x such that $f(x) = 0$.

A simple method consists of trying to find values of x where the function changes sign. We would then know that one solution lies somewhere between these values. For example: If $f(a) \times f(b) < 0$ and $a < b$ then the solution x must lie between these values: $a < x < b$. We could then try to narrow the range and hopefully converge on the true value of the root. This is the basis of the so-called bisection method.

The bisection method is an iterative scheme (repetition of a simple pattern) in which the interval is halved after each iteration to give the approximate location of the root. After i iterations the root (let's call it x_i , i.e., x after i iterations) must lie between

$$a_i < x_i \leq b_i$$

and an approximation for the root is given by:

$$p_i = \frac{(a_i + b_i)}{2}$$

where the error between the approximation and the true root, ϵ_i , is bounded by:

$$\epsilon_i = \pm \frac{(b_i - a_i)}{2} = \pm \frac{(b_1 - a_1)}{2^i}.$$

At each iteration the sign of the functions $f(a_i)$ and $f(p_i)$ are tested and **if** $f(a_i) \times f(p_i) < 0$ the root must lie in the half-range $a_i < x < p_i$. Alternatively the root lies in the other half (see figure). We can thus update the new lower and upper bound for the root:

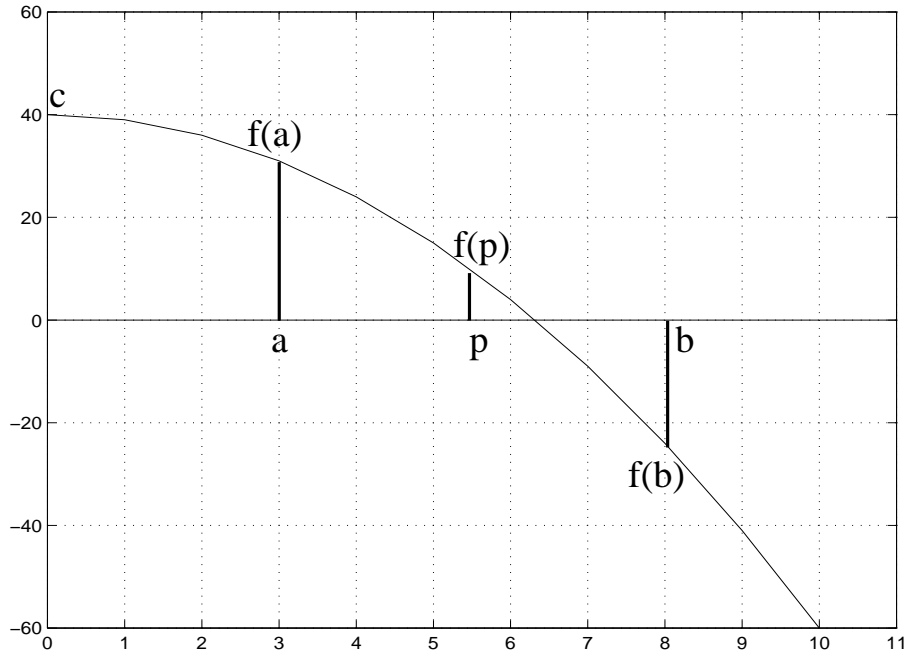
if $f(a_i) \times f(p_i) < 0$ then $a_{i+1} = a_i$ and $b_{i+1} = p_i$

else $a_{i+1} = p_i$ and $b_{i+1} = b_i$

The bisection method is guaranteed to converge to the true solution but is slow to converge since it uses only the sign of the function. An alternative is the Newton–Raphson method, which takes into account the gradient of the function, $f'(x)$, and only needs one starting guess for the root. In the Newton–Raphson method the next approximation to the root is given by:

$$p_{i+1} = p_i - \frac{f(p_i)}{f'(p_i)}.$$

D. Notes on algorithm and implementation



The square root of a number will be found by calling a user-defined function to implement one of the iterative algorithms (i.e., repetition of a pattern of actions) described above. Review

tutorial section 7, which describes how to define a function and how to pass parameters to the function and return values to the main program.

You are required to define a function to find the square root of a number c , i.e., find the zeroes of $f(x) = c - x^2$. Your solution is to be accurate to 5 decimal places. Since the number input by the user is between 1 and 100 the root will satisfy $0 < x \leq 10$ and a valid initial guess for the lower and upper bound for the solution will always be $a_1 = 0.1$ and $b_1 = 10.1$. The error after i iterations will be $\pm \frac{10}{2^i}$. To produce a solution that is accurate to 5 decimal places we will need more than 20 iterations.

1. Getting Started:

Start with a very simple program that prompts the user for the value of a real number in the range $1 < c \leq 100$. Alternatively, steal RandDemo6.cc's method of getting the float c from the command-line (something like `sscanf(..., "%f", &c)`).

2. Function definition

You are required to **define** a *function*, `MySquareRoot()`, which is *passed* the number (i.e., a single parameter of type `float`) and *returns* the approximate value of its square root (i.e., return value type is `float`).

The C++ code for the function should be placed below the body of `main()`. Begin the implementation of the function by typing in the *function header* and the opening and closing braces. For example:

```
float MySquareRoot(float square)
{
    // Body of function definition
}
```

3. Function body and implementation of algorithm

Inside the body of the function (i.e., after the opening brace):

- (a) You will need to declare *local variables* to store the values of a_i , b_i , p_i and $f(a_i) \times f(p_i)$. For example: `lower`, `upper`, `root` and `sign`. Initialize the values of `lower` and `upper` to 0.1 and 10.1 respectively. (A possible additional programming goal would be to make a smarter initialization method such that square roots can be found for any c in a larger range.)
- (b) Set up a loop using the `while` or `for` repetition control statements to repeat the following algorithm (bisection method) at least 20 times.
- (c) In each execution of the loop:
 - Estimate the value of the root as the average of the lower and upper bounds. Store this value in variable `root`.
 - Evaluate the function at the current value of lower (i.e., a_i) and at the current estimate of the root, (p_i).
 - Evaluate $f(a_i) \times f(p_i)$ and store this value in variable `sign`.
 - Depending on the value of `sign` update the lower and upper bounds by the bisection method described above.

(d) The function must end with a return statement to pass back the approximate value of the square root.

4. Function declaration:

Declare the function by including the function prototype before `main()`. Compile your program to make sure you have not made any typing or syntax errors.

5. Testing of function:

Call your function from your program with the (actual) parameter, e.g. `number`. The *return value* of the function is to be assigned to a variable (e.g. `squareRoot`) which should also be declared in `main()`:

```
squareRoot = MySquareRoot(number);
```

Test your function creatively.

6. Loop

Set up a loop in the main routine to *call* the function 10 times to calculate the square roots of the integers 1, 2, ... 10.

Set yourself an **additional programming task** along the lines of the preceding tasks, and solve it by yourself. Then submit your task statement and your solution to the course pigeonhole.

Put your task statement and solution file in a new folder (IB.4) inside your existing folder – like this (if your CRSID is jb007)

```
mkdir /ux/physics/part_1b/c++/pigeonhole/jb007/IB.4
cp README /ux/physics/part_1b/c++/pigeonhole/jb007/IB.4
cp blah.cc /ux/physics/part_1b/c++/pigeonhole/jb007/IB.4
```

Please do not put enormous files in the course pigeonhole – just small text files and .cc files.

Deadline: You are strongly encouraged to complete this work on time during week 8. See page 5 for the final deadlines.

Revision tips for the Michaelmas/Lent holiday

The aim of the Lent-term sessions will be not only to learn more about C++ but also to see how programming can help you understand physics better.

If you want more practice with C++, try setting yourself some simple tasks – write a general units converter, programs to output prime numbers, programs to calculate lowest common multiples or highest common factors. Test the program `ForStatistics.cc` on 39 and find inputs for which it gives incorrect answers; write an improved version. Write a function that computes binomial coefficients more reliably and accurately than the method on page 54. Remember that you can use the linux PWF system remotely. If you have a personal computer, set up a C++-programming environment for yourself – all the software used in this course is free software, available for any computer platform.

More programming concepts

The new programming tools we'll use in the Lent term are arrays (great for representing vectors, or lists of similar things); structures (great for organizing things that belong together, such as a particle's position, mass, and velocity, or a vector and its range of indices); and how to modularize your code.

We now cover the following topics.

arrays	how to allocate memory on the fly for things like vectors
modularizing	chopping up your code so you can reuse it elegantly
structures and packages	how to use structures in your elegant modules
formatted output	getting the number of decimal places that you want

9 Arrays

More realistic programs need to store and process a substantial number of items of data. The types of variable considered so far have been simple data types capable of holding a single value.

Arrays are a consecutive group of variables (memory locations) that all have the same type and share a common name. In many applications, arrays are used to represent vectors and matrices. We now describe a simple approach to arrays. A more elegant approach will be described in due course.

9.1 Declaration

An array is declared by writing the type, followed by the array name and **size** (number of elements in the array) surrounded by square brackets.

<i>type</i>	<i>array-name</i> [<i>number-elements</i>];
float	position[3];
int	count[100];
char	YourSurname[50];

The above examples declare `position` to be an array that has 3 elements which are real numbers (e.g., 3D vector). `YourSurname` contains 50 characters (e.g., storing a surname) and `count` is an array of 100 integer values.

An array can have any legal variable name but it cannot have the same name as an existing variable. The array's size is fixed when the array is declared and remains the same throughout the execution of the program.

9.2 Array elements and indexing

To refer to a particular element of the array we specify the name of the array and the *position number* or **index** of the particular element. Array elements are **counted from 0** and not 1. The first elements will always have position and index 0 and the last element will have index number (position) $N-1$ if the array has N elements.

The first elements in the arrays declared above are referred to by `position[0]`, `count[0]` and `YourSurname[0]` respectively. The last element of an array declared as `array[N]` with N elements is referred to by `array[N-1]`. The last elements of the example arrays above are therefore referred to by `position[2]`, `count[99]` and `YourSurname[49]` respectively.

The index of an array can also be an expression yielding an integer value.

9.3 Assigning values to array elements

Elements of an array can be treated like other variables but are identified by writing the name of the array followed by its index in square brackets. So for instance the statements:

```
marks[1] = 90.0;
scaled[1] = (marks[1] - mean)/deviation;
```

show how the second element of an array called `marks` is assigned a value and how the second element of the array `scaled` is assigned the result of a calculation using this element value.

What is useful about arrays is that they fit well with the use of loops. For example:

```
int count[100];
for(int i=0; i< 100; i++)
{
    count[i] = 0;
}
```

can be used to **initialise** the 100 elements of an integer array called `count` by setting them all to 0. The following program uses arrays and loops to calculate the scalar product of two vectors input by the user.

```

// ScalarProduct.cc
// Calculating the scalar product between vectors input by user

#include <iostream>
using namespace std;

int main()
{
    float vectorA[3], vectorB[3], scalar=0.0;

    // Get input vectors from user.
    cout << "Enter elements of first vector: " << endl;
    for(int i=0;i<3;i++)
    {
        cin >> vectorA[i];
    }
    cout << "Enter elements of second vector: " << endl;
    for(int i=0;i<3;i++)
    {
        cin >> vectorB[i];
    }

    // Calculate scalar product.
    for(int i=0;i<3;i++)
    {
        scalar = scalar + (vectorA[i] * vectorB[i]);
    }

    // Output result.
    cout << "The scalar product is " << scalar << endl;
    return 0;
}

```

Note: care must be taken never to try to assign array elements that are not defined. For example, there's nothing to stop you from including the statement `cin >> vectorA[723]` in your program. The program will compile and will execute, but the consequences of such assignments on the execution of the program are unpredictable and very difficult to detect and debug. It is left to the programmer to check that the array is big enough to hold all the values and that undefined elements are not read or assigned. Assigning array values outside the permitted range is probably one of the most common programming errors. An object-oriented approach to programming, which we'll discuss in due course, can reduce the risk of such programming errors. There are also debugging utilities, which can help spot memory mismanagement. We'll discuss these later in the course.

9.4 Passing arrays to functions

Function definition and prototype

To pass an array to a function, the array type and name is included in the formal parameter list (i.e., in the function definition and prototype parameter list). The following is a function header for a function that receives an array of real numbers.

```
void NormaliseData(float arrayName[], int arraySize)
```

Function call

In C++ the entire array is passed using the array name (and **no** square brackets). This looks similar to *pass by value* but is actually a pass by reference. The actual parameter passed to the function, the name of the array, is in fact the memory *address* of the first element of the array. The important point to note here is that (as with passing by reference) when you pass an array to a function you will be able to access and modify the values of any of its elements. Arrays cannot be passed by value.

In the following call to the `NormaliseData()` function, a 300 element array, `marks`, is passed to the function:

```
NormaliseData(marks, 300);
```

The function can read and change the value of any of the elements of the array.

`RotationMatrix.cc` on page 70 is a working example.

If you need to pass only a single element of the array and you want to take advantage of the simplicity and safety of passing by value, this can be done, for example, by having `marks[2]` as an actual parameter in the call to the function. For example in the following function:

```
float ProcessIndividual(float mark);          //Function prototype
scaledMark = ProcessIndividual(marks[2]);    //Function call
```

the call passes the value of `marks[2]`, which it stores as a local variable (`mark`). Changes to the value of the local variable will not affect the value in the calling function.

Another way to enforce safe passing of an array that should *not* be modified by a function is to declare the array `const` from the point of view of the function, as illustrated by this function:

```
void print1( const float array[] , int N ) {
    for ( int n = 0 ; n < N ; n ++ ) {
        cout << array[n] << " " ;
    }
    cout << endl;
}
```

9.5 Character arrays

A common use of the one-dimensional array structure is to create character strings. A character string is an array of type `char` that is terminated by the null terminator character, `'\0'`. The symbol `\0` looks like two characters but represents one. It is called an *escape sequence*.

A character array can be *initialised* on declaration by enclosing the characters in double quotes. The null character will be automatically appended.

```

// CharacterArray.cc
// Initialisation of a character array and passing to functions

#include <iostream>
using namespace std;

void PrintExtensionNumber(char phoneNumber[]);

int main()
{
    char phone[11];
    char prompt[] = "Enter telephone number: ";

    // Get phone number
    cout << prompt ;
    cin >> phone ;

    // If first two digits are 3 then it is a university number.
    if( phone[0]=='3' && phone[1]=='3')
    {
        cout << "University extension ";
        PrintExtensionNumber(phone);
    }
    else
    {
        cout << "Dial 9-" << phone[0];
        PrintExtensionNumber(phone);
    }

    return 0;
}

// Function to print out a phone number, ignoring the first digit.
void PrintExtensionNumber(char phoneNumber[])
{
    for(int i=1; phoneNumber[i] != '\0'; i++)
    {
        cout << phoneNumber[i];
    }
    cout << endl;
}

```

9.6 Multi-dimensional arrays

The arrays that we have looked at so far are all one-dimensional arrays, however, C++ allows multidimensional arrays. For example to declare a two-dimensional array to represent the data of a 3 by 4 matrix called `myMatrix` we could use the following statement and syntax:

```
float  myMatrix[3][4];
```

You may think of the element with indices *i* and *j*, `myMatrix[i][j]`, as the matrix element in the *i* row and *j* column, but remember that array indices always start from 0.

The following program illustrates the passing of arrays to and from functions. The function `ComputeMatrix()` assigns values to the elements of a 2 by 2 rotation matrix. The actual parameters passed are the name of the rotation matrix (i.e., memory address of the first element) and an expression to determine the rotation angle in radians. The element `matrix[0][1]` is the element of the first row and second column of the matrix.

The function `RotateCoordinates()` computes the coordinates of a point after transformation by the following equation:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

It is passed the name of the matrix and the name of the initial position vector and transformed position vector.

```

// RotationMatrix.cc
// Program to calculate coordinates after rotation

#include <iostream>
#include <cmath>
using namespace std;

void ComputeMatrix(float matrix[2][2], float angle);
void RotateCoordinates(float rot[2][2], float old[2], float transformed[2]);

int main()
{
    float angle, point[2], transformedPoint[2];
    float rotation[2][2];

    // Get angle of rotation and initial position from input.
    cout << "Enter magnitude of rotation in xy plane in degrees: " ;
    cin >> angle;
    cout << "Input x and y coordinates: " << endl;
    cin >> point[0] >> point[1];

    // Calculate coefficients of rotation matrix and transform point.
    // The value of pi is declared in math as M_PI.
    ComputeMatrix(rotation, (M_PI*angle/180.0));
    RotateCoordinates(rotation, point, transformedPoint);

    // Output result.
    cout << "The (x,y) coordinates in the rotated system are ";
    cout << transformedPoint[0] << " and " << transformedPoint[1] << endl;
    return 0;
}

void ComputeMatrix(float matrix[2][2], float angle)
{
    matrix[0][0] = cos(angle);
    matrix[0][1] = sin(angle);
    matrix[1][0] = -sin(angle);
    matrix[1][1] = cos(angle);
}

void RotateCoordinates(float rot[2][2], float old[2], float transformed[2])
{
    transformed[0] = (rot[0][0] * old[0]) + (rot[0][1] * old[1]);
    transformed[1] = (rot[1][0] * old[0]) + (rot[1][1] * old[1]);
}

```

9.7 Structures

Arrays are examples of data structures in which all the elements must be of the same type. C++ allows the user to define more general data structures, using the keyword `struct` to define a collection of related variables of any type, called a **structure**. For example:

```
struct StudentType{
    char   name[100];
    int    age;
    int    entryYear;
    float  marks[5];
    char   college[20];
};
```

defines a new data type called `StudentType`, which is made up of five *fields* or *data members*: two integers, an array of floating point numbers and two character arrays. The body of the structure definition is delineated by braces and must end with a semicolon. The definition is placed at the top of a program file, between include directives and the function prototypes.

Once a structure has been defined, the structure name can be used to declare objects of that type. This is analogous to declaring simple variables.

```
StudentType person;
StudentType firstYear[400];
```

declares the variable `person` and the one-dimensional array, `firstYear[400]` to be of type `StudentType`.

Data members of a structure are accessed and assigned values by specifying the field (data member) name using the *dot operator*, for example:

```
person.age = 19;
firstYear[205].entryYear = 1999;
```

Structures have the advantage that, by collecting related items together, they can be manipulated as single items. For example whole structures can be copied using an assignment statement (unlike arrays):

```
firstYear[24] = person;
```

and manipulated efficiently with repetition control statements:

```
for(int i=0; i< 400; i++)
{
    firstYear[i].entryYear = 1999;
}
```

9.8 Enumerated constants

There are many examples of data that are not inherently numeric. For example, the days of the week, months of the year, colours. We can refer to such data types by defining symbolic constants and using these symbolic constants in expressions in the program. For example:

```
const int Mon=0, Tue=1, Wed=2, Thu=3, Fri=4, Sat=5, Sun=6;
```

C++ provides a more convenient way for the user to define a new data type. The C++ **enumeration** statement:

```
enum Days {Thu, Fri, Sat, Sun, Mon, Tue, Wed};
```

creates a new variable type with legal values Thu, Fri, ..., Wed, which are in fact **symbolic constants** for 0, 1, ..., 6. The enumeration is simply assigning an integer to a symbolic constant. The definition makes it convenient to work with days of the week.

Variables can be declared as having the user-defined type and this will ensure that they are only assigned one of the legal values. The following statement declares the variable day to have the user-defined type, Days.

```
Days    day;
```

10 Reading and writing to files

Storage of data in variables and arrays is temporary. Files are used for permanent retention of large amounts of data. We will now show how C++ programs can process data from files. To perform file processing in C++, the header file `<fstream>` must be included. The latter includes the definitions of `ifstream` and `ofstream` *classes* (special structure definitions). These are used for input from a file and output to a file.

The following command opens the file called, `name.dat`, and reads in data, which it stores sequentially in variables `a` and `b`:

```
ifstream fin;
fin.open("name.dat");
fin >> a >> b;
```

The *object* called `fin` is declared to be of type (*class*) `ifstream`. The *member function* called `open()` is used to associate the file name, `name.dat`, with the object name, `fin`.

In a similar way we can write data (in this example, the values of an array) to a file called `output.dat` with:

```
ofstream fout;
fout.open("output.dat");
for(int i=0; i<N; i++)
{
    fout << array[i] << endl;
}
```

`fin` and `fout` are `ifstream` and `ofstream` *objects* respectively. Note that `fin` and `fout` are arbitrary names assigned by the programmer. Using `fin` and `fout` highlights the similarity with the simple input and output statements of section 5, which use the input and output stream objects `cin` and `cout` respectively. The syntax of the input and output statements is identical. The only difference is that care must be taken to ensure the files are opened and closed correctly, i.e., that the file exists and is readable or writable. This can be checked by testing the value of `fin.good()` or `fout.good()`. These will have the values `true` if the files are opened correctly.

The program on the next page reports an error if the file name specified was not found or could not be opened and prompts the user for another file name. After finishing reading from and writing to files, the files must be **closed** using:


```
fin.close();
fout.close();
```

and the `ifstream` and `ofstream` *objects* `fin` and `fout` can be re-assigned.

```
// OpenFile.cc
// Program to read data from a file. File name is input by user.

#include <iostream>
#include <fstream>
using namespace std;

int main()
{
    char fileName[80];

    // Get filename from keyboard input
    cout << "Enter input file name: ";
    cin >> fileName;

    // Declare fin to be of type (class) ifstream
    // Associate file name with fin
    ifstream fin;
    fin.open(fileName);

    // Prompt for new file name if not able to open
    while(fin.good() == false)
    {
        cout << "Unable to open file. Enter a new name: ";
        cin >> fileName;
        // once bad, the file stream will stay bad unless cleared
        fin.clear();
        fin.open(fileName);
    }
    return 0;
}
```

11 Arrays done better

Above, we declared arrays of fixed size like this

```
float          position[3];
int            count[100];
for(int i=0; i< 100; i++)
{
    count[i] = 0;
}
```

This fixed-size approach is an ugly way of doing things – it's inflexible, and it hard-wires numbers like '3' and '100' into your code, whereas such numbers should almost always be

parameters. It would be more elegant to replace the '100' above by a name (such as N), and use that name everywhere, and fix the value of N just once at the start of the program. It's even more elegant to allow N to be set interactively or on the command-line of the program – in the way that `RandDemo6.cc`, for example, could optionally set the number of points N (page 34 of tutorial 1). The ugly way of doing arrays does not allow the creation of arrays of size controlled by a variable.

In C++ we can create variable-sized arrays on the fly while a program is running. We use a special command `new` to allocate the memory for an array, and a complementary command `delete` to free up the memory again when we don't need it any more.

The example program `NewDelete.cc` illustrates how to use `new` and `delete`, and how to pass an array to a function.

```
// NewDelete.cc
#include <iostream>
using namespace std;

void show( double *a , int N ){
    for( int n=0; n<N; n++ )
        cout << "a["<<n<<"]=\t"
            << a[n]
            << endl ;
}

int main()
{
    double *a ;
    // This creates a pointer but doesn't allocate any memory for the array

    int    N = 20 ;

    a = new double[N] ; // allocates memory for a[0]..a[N-1]
    for( int n=0; n<N; n++ )
        a[n] = n*n ;
    show( a , N ) ;
    delete [] a ;      // frees the memory
}
```

In C++, array indices by default run from zero. But if you want an array that runs from, say, 1 to N , then you can do this by creating an array `a[0]...a[N-1]` of the right size, then offsetting the pointer by 1 (using `b=a+1`), so that `b[1]...b[N]` points to the same locations as `a[0]...a[N-1]`. This convenient offsetting is demonstrated in the next example.

We can also allocate memory on the fly for more complex objects such as matrices. A good way to think of an $M \times N$ matrix with typical entry `q[m][n]` is that it consists of M vectors, each of length N . We can allocate memory for *a pointer (q) to an array of pointers* (`q[1]...q[M]`). Each of the pointers `q[1]...q[M]` is just like the pointer `a` in the previous example, pointing to its own personal chunk of memory of size N . This way of handling matrices is demonstrated in several examples on the website.

You can pass arrays to functions only by reference, not by value.

12 Modularizing

In section 7, we introduced the idea of chopping a program into small chunks called functions. It's good style to put each conceptual bit of the program in its own function. Try to avoid writing the same piece of code more than once.

In accordance with this principle of writing everything once only, it's also a good idea to put any function that you might want to use again into a file that other programs can make use of. There's a couple of ways to split programs over multiple files.

1. The simple way with `#include`. If you put the directive `#include "blah.cc"` on one line of a C++ program then the compiler will behave exactly as if the contents of the file `blah.cc` were there, in your file, at that point. In this way you need to compile only one file – the other files get read in appropriately.
2. The professional way with *linking*. Alternatively, you can split the program into multiple files *each of which is compiled separately*. In this approach, the compiler needs to be run several times, once for each separate file, and then a final time to *link* the compiled files together. *Compiling* each individual `.cc` file creates a corresponding `.o` file. *Linking* takes all the `.o` files and combines them into a single executable. Just one of the `.cc` files should contain a definition of the `main` function, which is where the executable starts.

When compiling an individual file, the compiler doesn't need to know anything about the functions in the other files, except for the syntax of any functions that get used in the current file. That syntax is conveyed to the compiler by function declarations. The recommended technique to handle these function declarations is to ensure that all functions in the file `blah.cc` are *declared* in another file called `blah.h`, which is `#included` by `blah.cc` and by any other files that wish to use `blah.cc`'s functions.

This 'linking' technique is illustrated by the four files that follow. The main program is in `package1.cc`. This program uses utility functions for memory allocation and array printing, located in the file `tinyutils.cc`. Both `package1.cc` and `tinyutils.cc` include the header file `tinyutils.h`. And finally, to keep track of what needs compiling when, it is essential to use a `makefile`. The `makefile` must contain an explicit statement that "package1 depends on package1.o and tinyutils.o", and (on the following line, *which must start with a single tab character*) an explicit instruction of how to link them together.

```
// tinyutils.h
//   declares the functions defined in tinyutils.cc

using namespace std;

// memory management
double *dvector ( int low, int high ) ;
void   freedvector ( double* a , int low ) ;

// printing
void printVector( double *b , int lo , int hi , int style=1 ) ;
// Note that any default parameter values (such as 'style=1')
// must be specified in the declaration.

// maths
double square( double a ) ;
```

```
# makefile for package1 and package2

CFLAGS = -ansi -g -Wall

LIBS = -l stdc++ -lm

CXX = g++

# These lines define what package1 depends on, and how to make it
package1: package1.o tinyutils.o
$(CXX) $(CFLAGS) $(LIBS) package1.o tinyutils.o -o package1
package2: package2.o tinyutils2.o
$(CXX) $(CFLAGS) $(LIBS) package2.o tinyutils2.o -o package2

%.o: %.cc
$(CXX) $(CFLAGS)      $< -c -o $@

%: %.cc makefile
$(CXX) $(CFLAGS) $(LIBS) $< -o $@
```

```

// package1.cc
// demonstrates how to use functions
// defined in a separately-compiled file (tinyutils.cc)

#include <iostream>
using namespace std;

// Both this file and tinyutils.cc include the
// function declarations from a single header file:
#include "tinyutils.h"

// In this example, we use functions 'dvector',
// 'square', 'printVector', and 'freedvector', all defined in
// tinyutils.cc

int main()
{
    double *b , *a ;
    int     N = 20 ;

    // allocate the space for b[1]..b[N]
    b = dvector( 1 , N ) ;
    a = dvector( 1 , N ) ;

    for ( int n = 1 ; n <= N ; n ++ )
        a[n] = static_cast<double>(n) ;
    for ( int m = 1 ; m <= N ; m ++ )
        b[m] = square( a[m] ) ;
    printVector( b , 1 , N ) ;

    // free the memory
    freedvector( b , 1 ) ;
    freedvector( a , 1 ) ;
    return 0;
}

```

```

// tinyutils.cc
// provides functions for double vectors allocation and clean-up
#include <iostream>
using namespace std;
#include "tinyutils.h"

// allocates memory for an array of doubles, say b[low]..b[high]
// example usage:  b = dvector( 1 , N ) ;
double *dvector ( int low, int high ) {
    int N = high-low+1 ;
    double *a ;
    if ( N <= 0 ) {
        cerr << "Illegal range in dvector: "
              << low << ", " << high << endl ;
        return 0 ; // returns zero on failure.
    }
    a = new double[N] ; // allocates memory for a[0]..a[N-1]
    if(!a) {
        cerr << "Memory allocation failed\n" ;
        return 0 ; // returns zero on failure.
    }
    return (a-low) ; // offset the pointer by low.
} // the user uses b[low]..b[high]

void freedvector ( double *b , int low ) {
    delete [] &(b[low]) ; // The '[]' indicate that what's
} // being freed is an array

// Note that default parameter values (such as 'style=0') have already
// been specified in the function declaration in tinyutils.h.
void printVector( double * b , int lo , int hi , int style ) {
    // style 1 means "all on one line"; style 0 means "in one column"
    for ( int n = lo ; n <= hi ; n ++ ) {
        cout << b[n] ;
        if(style) {
            if ( n == hi )    cout << endl;
            else               cout << "\t" ;
        } else {
            cout << endl;
        }
    }
}

double square( double x ) {
    return x*x ;
}

```

When we type `make package1`, the following things happen.

1. `make` looks at the makefile and learns that `package1` depends on `package1.o` and `tinyutils.o`.
2. If `package1.o` needs to be made, `make` invokes

```
g++ -ansi -pedantic -g -Wall package1.cc -c -o package1.o
```

At this stage, the compiler compiles just the functions that are defined in `package1.cc`.

3. Similarly for `tinyutils.o`, `make` invokes

```
g++ -ansi -pedantic -g -Wall tinyutils.cc -c -o tinyutils.o
```

4. For the final linking step, `make` invokes

```
g++ -ansi -pedantic -g -Wall -l stdc++ -lm package1.o tinyutils.o -o package1
```

yielding the executable `package1`. It's at this stage that the compiler will complain if any functions have been declared but not defined.

13 Structures and packages

We described, last term, how to use structures. Structures are a great way to organize things that belong together, and that should never really be separated from each other, such as a vector and its index range. By putting such things together into a single object, we can make code briefer (because we just refer to the one object, rather than its parts), and less buggy. The next example shows how to rewrite the previous example's vector-creation and vector-printing using a structure that contains the vector's pointer and its index range. The structure is defined in the header file. Why is it a good idea to use this structure? For this toy example, it doesn't seem like a big deal, but what you can notice is that function-calls that do things with the vector, once it's been created (such as `printDvector(b)`), are simpler and briefer, because we don't need to send along the baggage (`low, high`) that is required in the un-structured approach. The structure contains this baggage, so when we pass the pointer to the structure to other functions, those functions get access to exactly the baggage they need. The only down-side of this structured approach is that when we want to access the contents of the vector, we have to get the pointer to the vector out of the structure, so what used to be `a[n]` in the old approach (where `a` was the pointer to the array) becomes `a.v[n]` in the new approach (where `a` is the structure).

```

// package2.cc
// demonstrates how to use structures and functions
// defined in a separately-compiled file (tinyutils2.cc).
// The structure Dvector is defined in tinyutils2.h

#include <iostream>
using namespace std;

// Both this file and tinyutils2.cc include the
// function declarations from a single header file:
#include "tinyutils2.h"

int main()
{
    Dvector a , b ;
    int     N = 20 ;

    // allocate the space for b.v[1]..b.v[N]
    allocate( b , 1 , N ) ;
    allocate( a , 1 , N ) ;

    for ( int n = 1 ; n <= N ; n ++ )
        a.v[n] = static_cast<double>(n) ;
    for ( int m = 1 ; m <= N ; m ++ )
        b.v[m] = square( a.v[m] ) ;
    printDvector( b ) ;

    // free the memory
    freeDvector( b ) ;
    freeDvector( a ) ;
    return 0;
}

```



```

// tinyutils2.h
//   declares the structures and functions defined in tinyutils2.cc

using namespace std;

struct Dvector {
    double *v ; // the vector itself
    int     low;
    int     high;
} ; // don't forget the semicolon in the structure definition

// memory management
int     allocate( Dvector &a , int low, int high ) ;
void     freeDvector ( Dvector &a );

// printing
void printDvector( Dvector &b , int style=0 ) ;
// Default parameter values (such as 'style=0')
// must be specified in the declaration.

// maths
double square( double a ) ;

```

```

// tinyutils2.cc
// provides functions for double vectors allocation and clean-up
#include <iostream>
using namespace std;
#include "tinyutils2.h"

// allocates memory for an array of doubles. Example: allocate( b, 1 , N ) ;
int allocate ( Dvector &a , int low, int high ) {
    a.low = low ; a.high = high ;
    int N = high-low+1 ;
    if ( N <= 0 ) {
        cerr << "Illegal range in dvector: "
              << low << ", " << high << endl ;
        return 0 ; // returns zero if failure.
    }
    a.v = new double[N] ; // allocates memory for a[0]..a[N-1]
    if(!a.v) {
        cerr << "Memory allocation failed\n" ;
        return 0 ;
    } else {
        a.v -= low ; // offset the pointer by low.
        return 1 ;
    }
}

void freeDvector ( Dvector &b ) {
    delete [] &(b.v[b.low]) ;
    b.high = b.low - 1 ;
}

// Note that default parameter values (such as 'style=0') have already
// been specified in the function declaration in tinyutils2.h.
void printDvector( Dvector &b , int style ) {
    // style 1 means "all on one line"; style 0 means "in one column"
    for ( int n = b.low ; n <= b.high ; n ++ ) {
        cout << b.v[n] ;
        if(style) {
            if ( n == b.high )      cout << endl;
            else                    cout << "\t" ;
        } else {
            cout << endl;
        }
    }
}

double square( double x ) {
    return x*x ;
}

```

14 Formatted output

We've mainly printed out numbers using `cout` with commands like

```
cout << myint << " " << mydouble << endl ;
```

What if you don't like the way `cout` makes your numbers look, however? Too many decimal places? Too few? Well, `cout` can be bossed around and can be told to serve up numbers with different numbers of decimal places. You can find out more about `cout` by looking in a manual online or on paper.

Here we'll describe another way to control output formatting, using the old-fashioned C function, `printf` (which means print, formatted). It's probably worth learning a bit about `printf` because its syntax is used in quite a few languages.

The commands

```
int age = 84; printf( "his age is %d years" , age ) ;
```

will print the string 'his age is 84 years'. The command

```
printf( "%d %d\n" , i , j ) ;
```

causes the integers `i` and `j` to be printed, separated by a single space, and followed by a newline (just like `cout << i << " " << j << endl ;`). The command

```
printf( "%3d %-3d\n" , i , j ) ;
```

causes the same numbers to be printed out but encourages `i` to take up 3 columns, and to be right-aligned within those 3 columns; and encourages `j` to take up 3 columns and to be left-aligned. Text and numbers can be mixed up. For example,

```
printf( "from %3d to %-3d\n" , i , j ) ;
```

might be rendered as

```
from 123 to 789
```

Notice how you specify the format of the whole string first, then the missing bits, each of which was indicated in the format string by a specifier, *%something*. The format specifier for a single character is `%c`; for a string of characters, `%s`. The special character `\n` is a newline; `\t` is a tab; `\"` gives a double quote; `\\` gives a single backslash. The command

```
printf( "%f %e %g \n" , x , y , z ) ;
```

prints the floating point numbers `x`, `y`, and `z` as follows: `x` is printed as a floating point number with a default number of digits (six) shown after the decimal point; `y` is printed in scientific (exponential) notation; `z` is printed using whichever makes more sense, fixed floating point or scientific notation. The program `Printf.cc` on the website illustrates more examples. I usually use the format

```
printf( "%12.6g %12.6g %12.6g" , x , y , z ) ;
```

to print my real numbers – this format gives each of them 12 columns, and shows 6 digits of precision.

Physics objectives: to better understand Newton's laws, circular motion, angular momentum, planets' orbits, Rutherford scattering, and questions about spacemen throwing tools while floating near space shuttles.

Computing objectives: structures, arrays, using gnuplot; simulation methods for differential equations (Euler, Leapfrog).

You are encouraged to work in pairs, with the weaker programmer doing all the typing.

Task

Simulate Newton's laws for a small planet moving near a massive sun. For ease of plotting, assume the planet and its velocity both lie in a two-dimensional plane. Put the sun at the origin $(0,0)$ and let the planet have mass m and initial location $\mathbf{x}^{(0)} = (x_1, x_2)$ and initial velocity $\mathbf{v}^{(0)} = (v_1, v_2)$. The equations of motion are:

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{v}(t) \\ m \frac{d\mathbf{v}}{dt} &= \mathbf{f}(\mathbf{x}, t), \end{aligned} \quad (1)$$

where, for a standard inverse-square law (gravity or electrodynamics) the force \mathbf{f} is:

$$\mathbf{f}(\mathbf{x}, t) = A \frac{\mathbf{x}}{\left(\sqrt{\sum_i x_i^2}\right)^3}, \quad (2)$$

with $A = -GMm$ for gravitation, and $A = Qq/(4\pi\epsilon_0)$ for electrodynamics with two charges Q and q .

Try to write your programs in such a way that it'll be easy to switch the force law from inverse-square to other force laws. For example, Hooke's law says:

$$\mathbf{f}(\mathbf{x}, t) = k\mathbf{x}. \quad (3)$$

How should we simulate Newton's laws (1) on a computer? One simple method called Euler's method makes *small, simultaneous* steps in \mathbf{x} and \mathbf{v} . We repeat lots of times the following block:

- EULER'S METHOD
- 1: Find $\mathbf{f} = \mathbf{f}(\mathbf{x}, t)$
 - 2: Increment \mathbf{x} by $\delta t \times \mathbf{v}$
 - 3: Increment \mathbf{v} by $\delta t \times \frac{1}{m}\mathbf{f}$
 - 4: Increment t by δt

We might hope that, for sufficiently small δt , the resulting state sequence (\mathbf{x}, \mathbf{v}) in the computer would be close to the true solution of the differential equations.

Euler's method is not the only way to approximate the equations of motion.

An equally simple algorithm can be obtained by reordering the updates. In the following block, the updates of \mathbf{f} and \mathbf{x} have been exchanged, so the force is evaluated at the new location, rather than the old.

- A: Increment \mathbf{x} by $\delta t \times \mathbf{v}$
- B: Find $\mathbf{f} = \mathbf{f}(\mathbf{x}, t)$
- 3: Increment \mathbf{v} by $\delta t \times \frac{1}{m}\mathbf{f}$
- 4: Increment t by δt

It's not at all obvious that this minor change should make much difference, but often it does. This second method, in which position and velocity are updated alternately, is called the Leapfrog method. Here is a precise statement of the Leapfrog method.

```

LEAPFROG METHOD
Repeat
{
    Increment  $\mathbf{x}$  by  $\frac{1}{2}\delta t \times \mathbf{v}$ 
    Increment  $t$  by  $\frac{1}{2}\delta t$ 
    Find  $\mathbf{f} = \mathbf{f}(\mathbf{x}, t)$ 
    Increment  $\mathbf{v}$  by  $\delta t \times \frac{1}{m}\mathbf{f}$ 
    Increment  $\mathbf{x}$  by  $\frac{1}{2}\delta t \times \mathbf{v}$ 
    Increment  $t$  by  $\frac{1}{2}\delta t$ 
}

```

In this version, we make a half-step of \mathbf{x} , a full-step of \mathbf{v} , and a half-step of \mathbf{x} . Since the end of every iteration except the last is followed by the beginning of the next iteration, this algorithm with its half-steps is identical to the previous version, except at the first and last iterations.

When simulating a system like this, there are some obvious quantities you should look at: the angular momentum, the kinetic energy, the potential energy, and the total energy.

Techniques to use:

1. Represent the vectors \mathbf{x} , \mathbf{v} , and \mathbf{f} using **arrays**. Make sure you look at some simple examples of arrays before using them for planets.
2. Put all the variables and arrays associated with a single object (such as a planet) in a **structure**. Make sure you look at some simple examples of structures.

What to do

This is a self-directed and self-assessed course, and I'd like you to choose a planet-simulating activity that interests you. Here are some suggestions. **You don't have to do all of these.** The more you do, the more educational it'll be. But do take your pick, and feel free to steal working code (e.g. from the course website) if you'd prefer to focus your energy on experimenting with working code, rather than writing and debugging your own.

1. Write code that implements Euler's method and the Leapfrog method. Get it going for one initial condition, such as $\mathbf{x}^{(0)} = (1.5, 2)$, $\mathbf{v}^{(0)} = (0.4, 0)$. Before running your program, predict the correct motion, roughly. Compare the two methods, using gnuplot to look at the resulting trajectories, the energies, and the angular momenta. (If you'd like more detailed guidance through a possible approach to this task, **see the appendix on p. 93**. A worked solution for this first part is also available.)
2. Once you have a trustworthy simulation: take a collection of similar initial conditions, all having the same initial position and initial energy but differing in the direction of the initial velocity. What happens? Show all the evolving trajectories in a single movie.

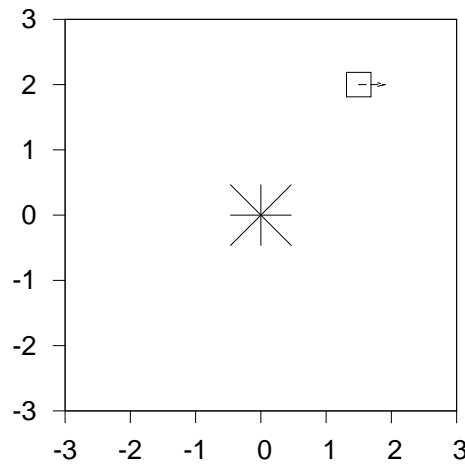


Figure 1: The initial condition $\mathbf{x}^{(0)} = (1.5, 2)$, $\mathbf{v}^{(0)} = (0.4, 0)$.

3. Or take initial conditions that differ slightly in their initial position or velocity, such as “the spaceshuttle and the spaceman who is 100 yards from the spaceshuttle, both orbiting the earth” (in what direction does he need to nudge himself in order to get home?). Or “the spaceman and the hammer” – if he throws his hammer ‘forwards’, where does it end up going? If he throws it ‘back’ or ‘sideways’, where does it end up going? (Here the idea is to simulate two planets orbiting a single sun; to get the spaceman analogy, think of the spaceman and his shuttle being like the two planets, and the earth playing the role of the sun.)
4. Get an initial condition that leads to a perfectly circular orbit. Now perturb that initial condition by giving a little kick of momentum in 8 different directions. Show all nine resulting trajectories in a single movie. Which kicks lead to the period of the orbit changing? Which kicks lead to the period of the orbit staying the same? Which kicks lead to orbits that come back to the place where the kick occurred? If an object is in an elliptical orbit, what kicks do you give it in order to make the orbit larger and circular? What kicks do you give it to make the orbit smaller and circular? What’s the best time, in an elliptical orbit, to give the particle a kick so as to get the particle onto an orbit that never comes back, assuming we want the momentum in the kick to be as small as possible?
5. Take initial conditions coming in from a long way away, particles travelling along equally-spaced parallel lines. What happens? (The distance of the initial line from the sun is called the impact parameter of the initial condition.)
6. People who criticise Euler’s method often recommend the *Runge–Kutta method*. Find out what Runge–Kutta is (Google knows), implement it, and compare with the Leapfrog method. Choose a big enough step-size δt so that you can see a difference between the methods. Given equal numbers of computational operations, does Runge–Kutta or Leapfrog do a better job of making an accurate trajectory? For very long runs, and again assuming equal numbers of computational operations, does Runge–Kutta or Leapfrog do a better job of energy conservation? Of angular momentum conservation?

Deadline: You are strongly encouraged to complete this work on time during week 1. See page 5 for the final deadlines.

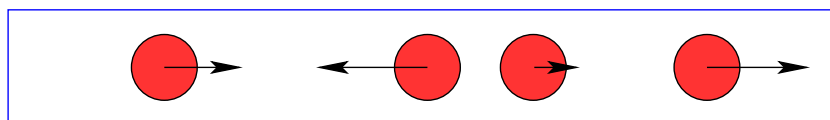
Physics objectives: to better understand collisions, conservation laws, and statistical physics, especially equipartition, properties of ideal gases, the concept of temperature, and the Boltzmann distribution; also the way in which microscopic physics leads to macroscopic phenomena; what happens when a piston compresses an ideal gas; adiabatic expansion; fluctuations and dissipation, equilibration of systems with different temperatures.

Computing objectives: structures, arrays, using gnuplot; memory allocation.

You are encouraged to work in pairs, with the weaker programmer doing all the typing.

Task

We're going to simulate hard spheres colliding with each other elastically in a box. An incredible range of interesting physics phenomena can be studied in this way.



The heart of this computing exercise is going to be a single function – let's call it `collide` – which receives two particles with masses m_1 and m_2 and velocities u_1 and u_2 , and returns the new velocities v_1 and v_2 of the two particles after an elastic collision between the two.

You could start by writing a function that implements this elastic collision. Check your function with special cases that you've solved by hand, and by putting in a range of initial conditions and evaluating whether total momentum and total energy are indeed conserved as they should be.

An elegant approach to this programming task uses a structure to represent each particle – something like this, for a particle moving in one dimension:

```
struct particle {  
    double x      ; // position  
    double p      ; // momentum  
    double im     ; // inverse mass  
    double v      ; // velocity  
    double T      ; // kinetic energy  
    double a      ; // radius of particle  
} ; // Note the definition of a structure ends with a semicolon
```

At this stage it's not crucial, but at some point I recommend making sure all references to the particle's mass use the *inverse mass*, rather than the mass – this allows you to treat the walls of the box as standard particles that just happen to have infinite mass (that is, inverse-mass zero).

Once you have defined a struct like `particle`, you may define an array of particles in just the same way that you define arrays of other objects like ints or doubles. For example

```
a = new particle[N] ;
```

What to do next

There's a lot of choice. This is a self-directed and self-assessed course, and I'd like you to choose a bonking-simulating activity that interests you.

Here are some suggestions. **You don't have to do all of these.** The more you do, the more educational it'll be. But do take your pick, and feel free to steal working code (e.g. from the course website) if you'd prefer to focus your energy on experimenting with working code, rather than writing and debugging your own.

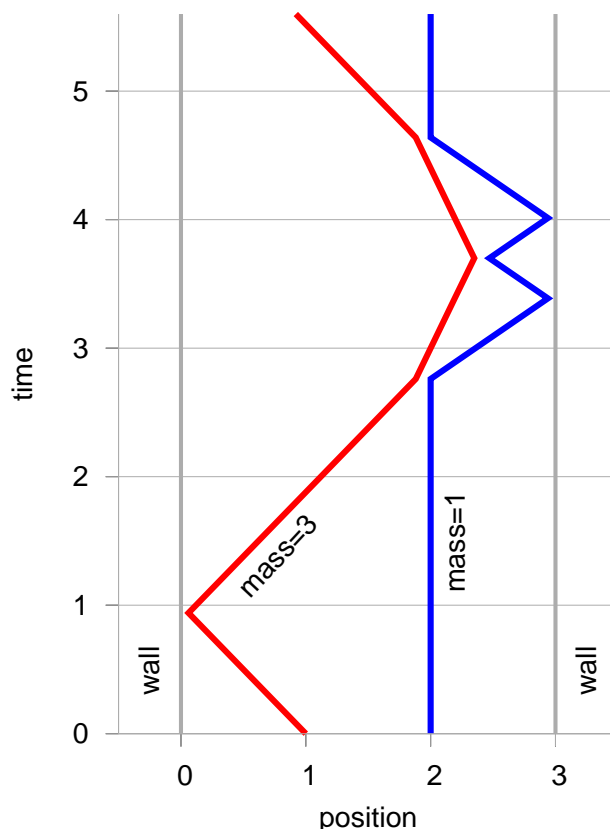
1. Write code that uses a one-dimensional `collide` function to simulate the motion of N particles in a one-dimensional box. Each particle can collide only with its immediate neighbours; each of the two end particles can collide with one wall too. To simulate the dynamics, you must identify the times at which collisions occur, and (assuming you want to make a realistic movie for showing in gnuplot) spit out the state of the simulation *at equally spaced times*. A suggested strategy is to take the current state and figure out which pair of adjacent particles will collide *next*. Then advance the simulation exactly to the moment of that next collision (stopping if appropriate at intermediate times along the way, so as to spit out the required states for the movie, equally spaced in time). Collisions should be handled by passing the pair of particles to the `collide` function. Motion in between collisions is simple (since there are no forces) and it should be handled by another function, `leapForward`, say. Printing out of the state at certain times of interest should be handled by another function, `showState`, say. (A worked solution for this first part is available on the course website.)

[A possible difficulty with this approach of computing all collisions that occur is that it is conceivable that the true number of collisions in a finite time might be very large, a phenomenon known as chattering. You can get the idea of chattering by imagining quickly squashing a moving ping-pong ball between a table-tennis bat and a table.]

2. Testing: Put just two particles in a box, with equal masses. Check that the dynamics are right. Make the two masses unequal. Make a scatter-plot of the positions of the two particles. Make a scatter-plot of the velocities of the two particles. Use more masses. Check that kinetic energy is conserved.
3. Put quite a few *unequal* masses in the box (say, 10 masses, with a variety of masses spanning a range of a factor of 4 in magnitude), run the simulation for a long time, and make histograms of the velocities of two of the particles whose masses are in the ratio 4:1. What do you find? Make histograms of the *positions* of the particles. If you make some of the particles *really heavy* compared to their neighbours, what happens to the histogram of the positions of the neighbours? For example, make all the particles except for the two end particles be much larger; or make half the particles (those on the left hand side) heavy, and the other half light. (In all simulations make sure no two adjacent particles have identical masses.)
4. What happens if the right-hand wall (with infinite mass) is moved at constant speed towards or away from the other wall?

Ye have heard it said that, under some circumstances, $pV^\gamma = \text{constant}$. What is γ for a *one-dimensional* ideal gas? How should the total energy vary with V under these conditions?

5. Set up N_1 light masses to the left of a single big heavy mass, and N_2 more light masses to the right of the heavy mass. Call the heavy mass a piston, if you like, and think of it as separating two ideal gases from each other. The light masses don't need to be identical to each other. An example set of masses for $N_1 = N_2 = 5$ could be (1.1, 1.2, 1.1, 1.3, 1.1, 100.0, 4.1, 4.2, 4.1, 4.8, 4.4) where the 100-mass is the piston. Give randomly chosen velocities to the particles. What should happen? How long does it take for 'equilibrium' to be reached? Give an enormous velocity to the piston and small velocities to the other particles. What should happen? How long does it take for 'equilibrium' to be reached? Can you get the piston to oscillate roughly sinusoidally (before 'equilibrium' is reached)? What is the frequency of such oscillations? Predict the frequency using the theory of adiabatic expansion/compression of gases.



The first six collisions between two particles of masses 3 and 1 and two walls.

If writing lots of files, or big files, for plotting with gnuplot, it may be a good idea to write them to the /tmp filesystem, which is a local filesystem on your machine. Files in /tmp are usually deleted when the computer reboots, so copy any important outputs into your normal filesystem. You can find the size of the /tmp file system with the unix command `df /tmp` (df means 'disc free'). *Don't ever expect to find your files in /tmp next time you log in.*

Deadline: You are strongly encouraged to complete this work on time during week 2. See page 5 for the final deadlines.

Part IB Computing Course Session 7: RECURSION

This is an optional extra.

Physics objectives: to better understand statistical physics by counting some interesting things.

Computing objectives: recursion.

Recursion means defining a function f in terms of the function f . For example we could define the factorial function $f(x)$ by:

1. if $x > 1$ then return $x \times f(x - 1)$
2. otherwise return 1.

A recursive function in a computer program is one that calls itself. Here's an example, following the above definition closely.

```
// factorial calculator - recursive
#include <iostream>
using namespace std;

int factorial (int a)
{
    if (a > 1)
        return (a * factorial (a-1));
    else
        return (1);
}

int main ()
{
    int number;
    cout << "Please type a number: ";
    cin >> number;
    cout << number << "! = " << factorial(number) << endl;
    return 0;
}
```

Errors in the definition of recursive functions often lead to disaster, since it's all too easy to write a function that keeps calling itself and never stops.

Some programmers find recursion an elegant way to express many programming tasks. Here is an example. The task is 'print all ternary strings of length L '. For $L = 1$ the output should be

0,1,2.

For $L = 2$ the output should be

00,01,02,10,11,12,20,21,22

Here is a solution.

```

// allStrings.cc
// Enumerate all ternary strings by recursion

#include <iostream>
using namespace std;

void appendAllStrings( char *prefix , int remainingLength )
{
    if (remainingLength == 0)
        cout << prefix << endl ;
    else {
        int lp = strlen(prefix) ;
        for ( int i = 0 ; i <= 2 ; i ++ ) { // Extend prefix by one character.
            prefix[lp] = i+'0' ;           // By adding i to the character '0'
                                           // we get the characters '0', '1', '2'
            appendAllStrings( prefix , remainingLength-1 );
        }
        prefix[lp] = '\0' ;               // Remove what was added
                                           // [ '\0' is the null character ]
    }
    return ;
}

int main ()
{
    int length;
    cout << "Please type a length: ";
    cin >> length;
    char *prefix ;
    prefix = new char[length] ;           // Assume this memory is all-null
    appendAllStrings( prefix , length ) ;
    delete [] prefix ;                   // Free the memory
    return 0;
}

```

The function `strlen` returns the length of the string generated so far; the line

```
prefix[lp] = ...
```

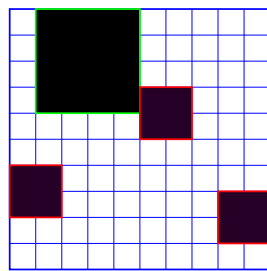
extends the string by one character. The algorithm proceeds by starting from the null string, and extending it by all possible single characters, extending each in turn by all possible characters, and so forth.

Such 'branching processes' are the type of problem for which recursion is especially recommended.

Recursion exercises

Solve each of these tasks using a recursive function.

1. Write a function called `twos` that takes a single integer as its argument and returns the number of factors of 2 in the number. (Hint: odd numbers have no factors of 2, numbers that are twice an odd number have one, numbers that are four times an odd have two, and so on.) For example: `twos(-12)`; should return 2.
2. Write a function called `printWithCommas` that takes a single nonnegative long integer argument and displays it with commas inserted in the conventional way. For example:
 - `printWithCommas(12045670)`; displays 12,045,670.
 - `printWithCommas(1)`; displays 1.
3. INTERESTING PROPERTIES OF A HARD SPHERE GAS.



A legal state of four particles in a 10×10 box.

A square box of size $H \times H$ lattice-points contains one big particle and $T = 3$ little particles. The big particle occupies 4×4 lattice points. Each little particle occupies 2×2 lattice points. Particles may not overlap. All legal states of this $T + 1$ -particle system are equiprobable. How probable are the alternative locations of the big particle? Find the answer for $H = 10$ (by recursively enumerating all legal states, and keeping count). It is a good idea to spit out the answer for smaller values of T along the way towards the answer for the biggest value of T .

If not all locations of the big particle are equiprobable then we can describe the effect of the little particles in terms of an effective ‘force’ acting on the big particle. Such forces are called ‘entropic forces’. Entropic forces in hard-sphere mixtures with a variety of sizes of spheres are an area of industrial research interest.²

²Y. Mao, P. Bladon, H. N. W. Lekkerkerker, and M. E. Cates. *Mol. Phys.* **92**, 151 (1997).

R. Dickman, P. Attard, and V. Simonian. ‘Entropic Forces in Binary Hard-Sphere Mixtures: Theory and Simulation’, *J. Chem. Phys.* **107**, 205–213 (1997).

‘Entropic Attraction and Repulsion in Binary Colloids Probed with a Line Optical Tweezer’ J. C. Crocker, J. A. Matteo, A. D. Dinsmore, and A. G. Yodh. *Physical Review Letters*, **82**, 4352–4355 (1999).

Appendices

PLANET: step by step guidance

1. Chop the problem into small pieces.
2. Define a structure that contains the state of the planet. (See page 79 of this tutorial and page 60 of the first term's tutorial.) Your structure could be as simple as:

```
struct particle {  
    double x[2] ; // (x,y) coordinates  
    double v[2] ; // velocity  
} ;
```

but as you continue, you will probably think of other sensible things to add to the structure. You might find it elegant to give a name to the dimensionality of the position space, say D.

```
#define D 2 // number of dimensions  
struct particle {  
    double x[D] ; // (x,y) coordinates  
    double v[D] ; // velocity  
} ; // Note the definition of a structure ends with a semicolon
```

If you then use D everywhere, it makes the meaning of your code clearer (compared with using '2'), and it makes it easier to update your simulation's dimension (say from 2 to 3 dimensions).

3. Write a simple main that defines a particle, and check that it compiles and runs.

```
int main()  
{  
    particle a ;  
  
    a.v[0] = 0.4;  
    a.v[1] = 0.0;  
    a.x[0] = 1.5;  
    a.x[1] = 2;  
  
    return 0;  
}
```

4. Write a function `showState` that prints out the particle's position and velocity. Call it from `main` and check that your program compiles and runs correctly. Here is an example of a function that does the right sort of thing:

```
void showState ( particle &a )
{
    int i=0;
    cout << "some of the state: " << a.x[i] << endl ;
}
```


This could be called by `main` using a command such as

```
showState( a ) ;
```

Notice the use of the ampersand in the function definition

```
void showState ( particle &a ).
```

This ampersand means that we are passing the particle by reference, rather than by value. (See page 45 of last term's tutorial.)

5. Remember the tips from last term about debugging your code. (1) Use a `makefile`. (2) Switch on all the compiler warnings. (3) Whenever you think that your code ought to compile, check that it does so. (4) If you have code that compiles ok, but that doesn't behave as expected when you run it, *run it in a debugger* (for example, `kdbg`). Even if it seems to be running as expected, it might be a good idea to run it in a debugger. When I use `kdbg`, the main things I click on are 'set breakpoint' (right click), 'run', and the 'step over'  icon.
6. Write a function that computes the squared-distance from the origin.
7. Write a function `Force` that computes the force acting on the particle, or the acceleration of the particle. (Perhaps the force or acceleration should be in your structure?)
8. Write a function `PositionStep` that makes a change of a particle's position in the direction of its velocity. One of the arguments of this function should be the required time step size.
9. Write a function `VelocityStep` that makes a change of a particle's velocity in the direction of its acceleration.
10. Write a function that computes the energy (kinetic and potential) of the particle.
11. Write a function that computes the angular momentum about the origin.
12. Write a Leapfrog simulator with a loop that calls the above functions appropriately.
13. Write the simulated trajectory to a file using a function like `showState`. To plot the trajectory from a file whose columns are

time, x_1 , x_2 , v_1 , v_2 ,

the following gnuplot commands may be useful.

```
set size ratio -1 ; ## this makes units on the x and y axes have equal size
plot 'tmp' u 2:3 w l, 'tmp' every 10 u 2:3:4:5 w vector
## this plots (x1,x2) with lines, and plots every 10th state using a vector
## of length (v1,v2) based at the point (x1,x2)
```

gnuplot's plot command is very versatile. The website shows how you can make a gnuplot animation using a single data file containing a trajectory.

14. Be aware of the size of the files you are writing. If you give gnuplot a file of more than 1 megabyte, you should expect it to be sluggish. Maybe it would be a good idea to reduce the amount of information written to file. You may be able to get better performance by using the local filesystem of the machine at which you are sitting. You can write anything you want into the folder /tmp. For example,

```
ls > /tmp/mylist
```

The /tmp folder is a good place to put anything large and anything that you want to access frequently or quickly. But don't leave anything important in /tmp – after you log out, the /tmp folder may be cleaned up.

A An introduction to object-oriented programming and classes

One of the most important differences between the C++ programming language and some other programming languages is the emphasis on the representation of data using programmer or user-defined data types. In C++ an extension to the definition of structures allows the user to include both *data members* (as above) and *member functions* which are allowed to process the data. The encapsulation of data and functions into packages called *objects* of user-defined types called *classes* is a key part of object-oriented programming.

In its simplest form, a class is like a structure that includes the definition of functions (member functions or class methods) that are allowed to process the data. See the simple example below.

1. Classes and Objects

The class (e.g. Date in the example) can be considered as a specification while the actual item (e.g. today) is an instance of a class and is called an *object*. Declaring an object is very similar to declaring a variable:

```
class-name object-name ;
```

2. Accessing data members and functions

The data members and member functions of the class can be accessed by simply naming them in conjunction with the name of the object they belong to using the *dot operator*.

```
object-name.item-name
```

3. Defining a class member function

The declaration and definition of class member functions is similar to those used for standard functions: we use function prototypes to declare the functions and place the statements to be executed in a function definition. The only difference is that with a class member function we must tell the compiler which class the function is a member of. This is done by including the name of the class in the function header using a *scope resolution* operator represented by `::`.

```
return-type class-name :: function-name (parameter-list )
```

4. Restricting access to class members

One of three levels of access can be specified for each class member (both data and functions) using the keywords `public`, `private` or `protected` followed by a colon. These refer to how the class members may be accessed in a program.

The `public` members of a class may be accessed directly from anywhere in the program that has access to an object of the class. The `private` members can be read or modified only by class member functions.

A simple example

This example shows how to create a simple class and define the class's member functions.

```
// SimpleClass.cc
// A program to demonstrate creation and use of a simple class for dates
#include <iostream>
using namespace std;

// Declaration of Date class
class Date {
public:
    void set(int, int, int);
    void print();

private:
    int year;
    int month;
    int day;
};

int main()
{
    // Create a Date object called today
    Date today;

    // Call Date member function set()
    today.set(1,9,1999);
    cout << "This program was written on ";
    today.print();
    cout << endl;
    return 0;
}

// Date member function definitions
void Date::set(int d, int m, int y)
{
    if(d>0 && d<31) day = d;
    if(m>0 && m<13) month = m;
    if(y>0) year =y;
}

void Date::print()
{
    cout << day << "-" << month << "-" << year << endl;
}
```

B Further reading

This tutorial has introduced the basic elements of the C++ programming language. The following references provide a comprehensive treatment and useful tips on good programming practice.

1. *C++ How to Program (5th edition)*, Deitel, H.M and Deitel, P.J.
Prentice Hall, Englewood (NJ), 2005.
2. *Code Complete: A Practical Handbook of Software Construction*, McConnell, S.
Microsoft Press, Redmond (WA), 1993.
3. *C++ in plain English*, Brian Overland.
M&T Books, 1999.