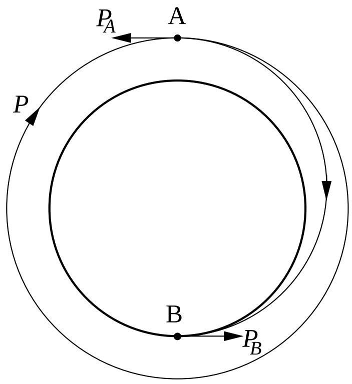
Part III Dynamics Examples **D.J.C. MacKay**

1. Orbits

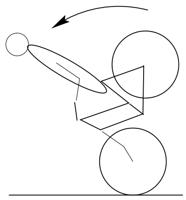


1.1 Write down the relevant conservation laws for a point mass moving under the influence of a central, conservative force. Use them to derive a differential equation for the radial motion.

1.2 Discuss the stability and closure of almost-circular orbits as a function of n for an attractive central force $F \propto r^n$. [Discuss at least the cases $n = 1$, $n = -1$, and $n = -2$.]

1.3 A lunar excursion module is initially in a circular orbit at a height $R/4$ above the lunar surface, where R is the radius of the moon. The objective is to land at point B by firing the module's rockets briefly at points A and B as indicated. Find the required impulses P_A and P_B, in terms of the initial momentum P of the module. (Ignore the rotation of the moon.) [*Ans:* $P_A = 0.057P$, $P_B = 1.18P$.]

2. Rigid bodies



2.1 Describe the forces acting on the two wheels when a moving cyclist applies (a) the front brakes or (b) the rear brakes. Which is the more effective for rapid deceleration?

2.2 A moving cyclist jams on her front brakes. Estimate how fast she needs to be going in order to roll right over, assuming that the front wheel does not skid.

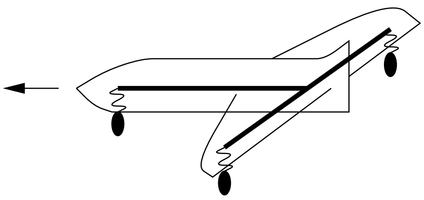
2.3 What happens when the *rear* brakes only are jammed on? At what speed is there a danger of hurtling over the front wheel?

3. Normal modes

3.1 Explain what is meant by the *normal modes* of oscillation of a many-particle system, and how their frequencies can be found. Discuss the relationship between *symmetries* of a system and its normal modes.

3.2 Discuss why the specific heats of gases at moderately high temperatures are in the sequence $H_2 < O_2 < H_2O < CO_2$.

3.3 An aircraft taking off is represented by two identical thin rods joined rigidly in a T configuration, with landing wheels attached to the ends by identical springs.



(i) Show that the normal mode frequencies ω are given by

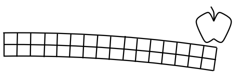
$$\omega^2 = 12\frac{k}{M}\,, \quad \frac{24}{5}\left(1 \pm \frac{1}{\sqrt{6}}\right)\frac{k}{M}$$

where k is the spring constant of each spring and M is the mass of the aircraft.

(ii) Describe the oscillations excited when (a) the front wheel, (b) a side wheel passes over a bump of height h in the runway. Assume that $\omega\tau \ll 1$, where τ is the time taken to go over the bump.

4. Elasticity

4.1 **Estimate** the Young's modulus of steel. Make two estimates, one based on empirical experience of steel, and one based on an atomic model. [Hints: what is the binding energy per atom of a generic chemical? What is the bond length? Roughly how far apart do the atoms need to be stretched to become unbonded?]



4.2 Estimate the deflection of a steel 30 cm ruler, 1 mm thick, when an apple (weight = 1 Newton) is placed on its free end, the other end being clamped horizontal. [The ruler is shown in side view in the figure.] Assume that the upper half of the ruler is stretched and the lower half is compressed. How does your answer depend on the ruler's thickness?